CONSTITUTIVE MODELING OF RUBBER COMPONENTS UNDER SMALL VIBRATION SUPERIMPOSED ON LARGE STATIC DEFORMATION CONSIDERING STRAIN-DEPENDENT PROPERTIES

Ji-Hyun Cho[*] , Sung-Kie Youn[*] , Wan-Sul Lee[*] and Bong-Kyu Kim[**]

[*]*Department of Mechanical Engineering, KAIST, Daejeon, Korea*
[**]*Hyundai Motor Company, Korea*

Summary A steady-state viscoelastic constitutive equation of filled rubber considering the effects of large static pre-strain and dynamic strain amplitude is proposed. The proposed model is based on the linearization of Simo's finite viscoelastic model and is modified to consider the influence of static pre-deformation and the dynamic strain dependent properties. Various dynamic tests are executed in order to get the model parameters and verify the model. The FEA results using the proposed model are compared with the test results to estimate the performance of the model.

INTRODUCTION

Many rubber components that are used as vibration isolator experience small oscillatory loads superimposed on large static deformation. The widely used Morman's model [1] is derived from the assumption that the time and large pre-strain effects are separable. It is observed in experiments that the separability assumption is restrictively applicable to unfilled rubber and the relaxation function of filled rubber is a function of pre-strain. Therefore it is very important to consider the influences of pre-strain in the constitutive theory of small viscoelastic motion superimposed on large static deformation in many engineering rubber materials. In the author's previous work [2], the K-Y viscoelastic constitutive model including the static deformation influence factor was proposed as a steady-state constitutive equation of filled rubber. Moreover, the dynamic strain amplitude affects the relaxation function of filled rubber and this behaviour is called as Payne's effect [3]. In this work, the viscoelastic constitutive model considering not only the static pre-strain but also the dynamic strain-dependent properties is proposed.

CONSTITUTIVE MODEL

The K-Y viscoelastic constitutive model for small vibration superimposed on large static deformation is derived through the linearization of Simo's finite viscoelastic model [4] and reference configuration transformation. The complex form of linearized model that relate the stress increment ΔS^* and the dynamic strain ε^* is written as follows.

(1) $$\Delta S^* = \left(J\frac{\partial^2 U}{\partial J^2} + P \right)\varepsilon^*_{kk} I - 2P\varepsilon^* - \frac{2}{3}\left[dev(\hat{\sigma}) \otimes I + I \otimes dev(\hat{\sigma}) \right] : \varepsilon^* + \left(1 + i\omega g^*\right)\mathscr{C} : \varepsilon^*$$

In the above equation, $\mathscr{C}$ is the Truesdell elasticity tensor. The relation between g^* and complex shear modulus $G^* = G' + iG''$ is

(2) $$\omega g^* = \frac{G''}{G_\infty} + \left(1 - \frac{G'}{G_\infty}\right) i \cdot$$

It is assumed that g^*, which represents the time effects, is not affected by static deformation. This separability assumption is suitable for the rubber which does not contain filler such as carbon black. However for filled rubber it is known by experiments that g^* depend on the static deformation. A static deformation influence factor, $c^*\left(B_0\right)$, is introduced to the constitutive equation in order to describe the non-separability nature of filled rubber. Also the function of strain amplitude and frequency is adapted in order to describe the effects of dynamic strain amplitude and frequency. $\hat{g}^*$,which is the correction of $1 + i\omega g^*$ in eqn. (1), is defined by

(3) $$\hat{g}^* = \tilde{g}^*(\omega, \varepsilon^*)c^*\left(B_0\right)$$

where c^* is a complex valued function that depends on the static deformation. Since the value of c^* is unity without the static deformation, following polynomial forms can be served as the static deformation influence factor.

(4) $$c_\gamma^*\left(\bar{I}_\gamma\right) = c_\gamma \; e^{i\theta_\gamma}, \; c_\gamma\left(\bar{I}_\gamma\right) = 1 + z_{\gamma 1}\bar{I}_\gamma + z_{\gamma 2}\bar{I}_\gamma^{\,2}, \; \theta_\gamma\left(\bar{I}_\gamma\right) = {}_\theta z_\gamma \, \bar{I}_\gamma$$

where generalized octahedral shear strain $\bar{I}_\gamma$ is defined as $\bar{I}_\gamma = \frac{1}{6}\left(2\,\bar{I}_1^2 - 6\,\bar{I}_2\right)^{1/2}$. $\bar{I}_\gamma$ is an invariant of B_0 and is reduced to octahedral shear strain under infinitesimal deformation.

Where $\tilde{g}^*(\omega,\varepsilon^*)$ is a complex valued function that describes the effects of dynamic strain amplitude and frequency. If it is assumed that the effects of frequency and strain dependent properties are separable, $\tilde{g}^*(\omega,\varepsilon^*)$ can be written by

(5) $$\tilde{g}^*(\omega,\varepsilon^*) = \tilde{g}_\omega^*(\omega)\tilde{g}_\varepsilon^*(\varepsilon^*)$$

(6) $$\tilde{g}_\omega^*(\omega) = f^a$$

(7) $$\tilde{g}_\varepsilon^*(\varepsilon^*) = \left(G'_\infty + \frac{\Delta G'}{1+(\|\varepsilon^*\|/\varepsilon_0)^{2m}}\right) + i\left(G''_\infty + \frac{2(G''_m - G''_\infty)(\|\varepsilon^*\|/\varepsilon_0)^m}{1+(\|\varepsilon^*\|/\varepsilon_0)^{2m}}\right)$$

where $\tilde{g}_\varepsilon^*(\varepsilon^*)$ is well known by KRAUS model [3]. $\tilde{g}_\omega^*(\omega)$ describes the effect of frequency. Eqn. (6) is the simple case of many other candidates. It is not the complete form, so some modifications are needed.

EXPERIMENT AND RESULTS

Dynamic tests for uniaxial tension and complex stress state are carried out in order to verify the proposed constitutive model. The test specimens are made of natural rubber filled with 50phr of carbon black. Fig 1 shows that eqn. (5)~(7) describe the effect of frequency and strain dependent properties well in the case of tension test results. Material parameters are determined from tension test results. Using these parameters, complex stress state test results are compared with the FEA results of the proposed model. Fig 2 shows that the proposed model shows good agreements with the test results of complex stress state in the case of fixed frequency and no pre deformation.

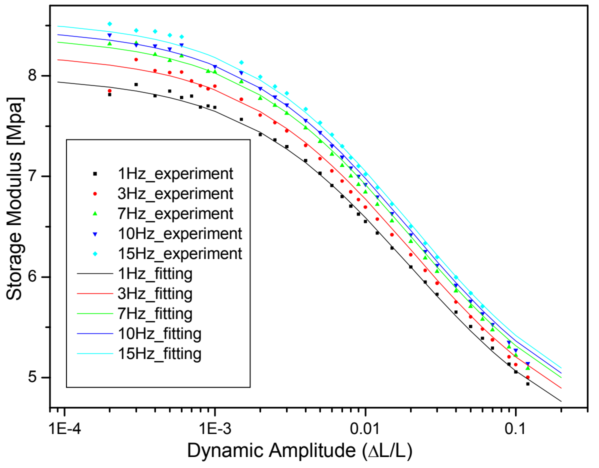

Fig 1 Curve fitting of tension test results

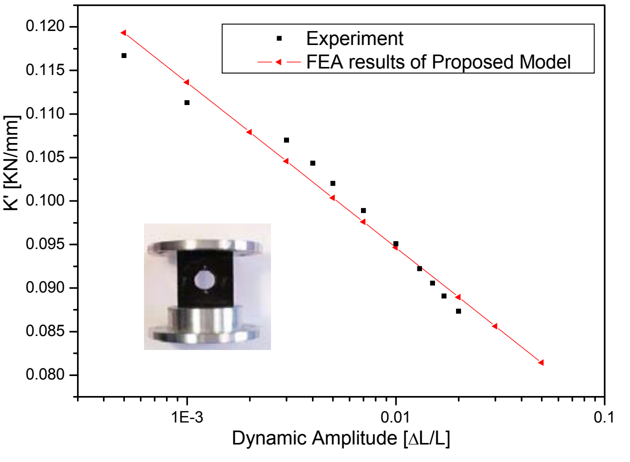

Fig 2 Comparison the experiment results with the FEA results of proposed model for complex shape test

CONCLUSION

A steady-state viscoelastic constitutive equation of filled rubber considering the effects of large static pre-strain and dynamic strain amplitude is proposed. Various dynamic tests are executed in order to get the model parameters and verify the model. The FEA results using the proposed model are compared with the test results to estimate the performance of the model. The proposed model shows good agreements with the test results of complex stress state.

References

[1] J. I. Sulivan, K. N. Morman and R.A. Pett : A Non-linear Viscoelastic Characterization of a Natural Rubber Gum Vulcanizate. Rubber Chemistry and Technology **53** : 805–822, 1980.

[2] B.-K. Kim and S.-K. Youn : A viscoelastic constitutive model of rubber under small oscillatory loads superimposed on large static deformation . Arch. Appl. Mech. **71** : 748–763, 2001.

[3] J.D.Ulmer : Strain dependence of dynamic mechanical properties of carbon black-filled rubber compounds. Rubber Chemistry and Technology **69** : 15-47,1996.

[4] J.C. Simo : Comput. A Fully Three-dimensional Finite-strain Viscoelastic Damage Model:Formulation and Computational Aspects. Method Appl. M. **60** : 153–173, 1987

Acknowledgement This work was supported by grant R01-2001-00393 from the Korea Science & Engineering Foundation.