MATHEMATICAL MODELS AND SIMULATION OF STICK-SLIP PROCESSES IN A CAR STEERING SYSTEM

Zbigniew Lozia*, Dariusz Zardecki **
* Warsaw Univ. of Technology, Faculty of Transport, Koszykowa75, PL 00-662 Warszawa, Poland
** Automotive Industry Institute, Jagiellonska 55, PL 03-301 Warszawa, Poland

Summary. *A method based on piecewise linear luz(...) and tar(...) projections was used for mathematical modelling and digital simulation of stick-slip phenomenon in a car-steering system. Results of simulations refer to typical open-loop tests.*

INTRODUCTION

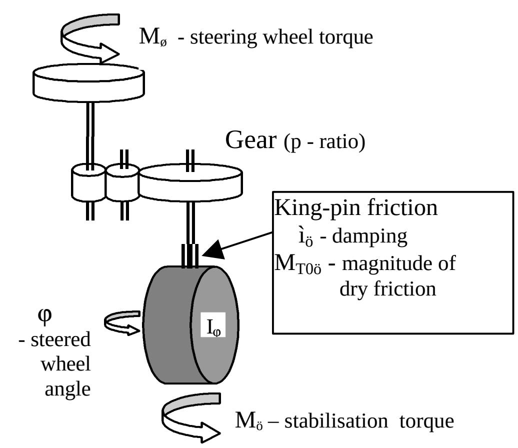
Mathematical modelling of dynamic systems with dry friction and digital simulation of the stick-slip phenomenon have been the subjects of non-linear dynamics presented in numerous research works (see state-of-the-art papers [1], [2], [3]). Surprisingly problems of stick-slip in a car steering system have been rarely taken into account. Dry friction (especially at the king-pin) could be large and it had a significant influence on the car active-safety. The piecewise linear luz(…) and tar(…) projections ([11], [12]) might be useful in studying these rather difficult theoretical problems (see [6]) and developing the complex simulation of car-dynamics with the stick-slip effects taken into consideration ([9], [13]).

CONCEPT OF MATHEMATICAL MODELING

Because of the dynamic properties of a car, its complex dynamic model could be seen as a cascade of two coupled autonomic partial models – a model of steering system dynamics and a model of vehicle motion dynamics.

Model of steering system dynamics

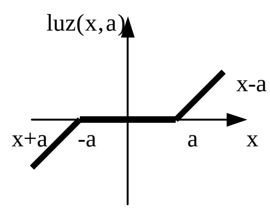
The piecewise-linear non-linear multi-body model of steering system dynamics has been used in the research. It has taken dry (kinetic and static) friction at king-pins as well as a steering box gear freeplay into consideration. This model could be useful for description of the dynamics of car steering system (both manual and power steering system types for 2WS as well as 4WS could be investigated). An idea of application of the luz(...) and tar(...) projections in the steering system mechanism model with dry friction is shown below. The simplest example of a single mass version is presented. (Note: No implicit description of stiction has been made!).

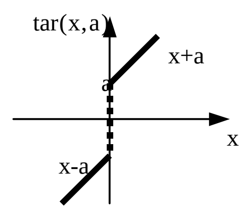


$$I_\varphi \cdot \ddot{\varphi}(t) = \begin{cases} -\mu_\varphi \cdot \mathrm{tar}\left(\dot{\varphi}(t), \dfrac{M_{T0\varphi}}{\mu_\varphi}\right) + pM_\psi(t) + M_\varphi & \text{when} \quad \dot{\varphi}(t) \neq 0 \\ \mathrm{luz}\left(pM_\psi(t) + M_\varphi(t), M_{T0\varphi}\right) & \text{when} \quad \dot{\varphi}(t) = 0 \end{cases}$$

where

$$\mathrm{luz}(x,a) = x + \frac{|x-a| - |x+a|}{2} \qquad \mathrm{tar}(x,a) = \mathrm{luz}^{-1}(x,a) \qquad a \geq 0$$





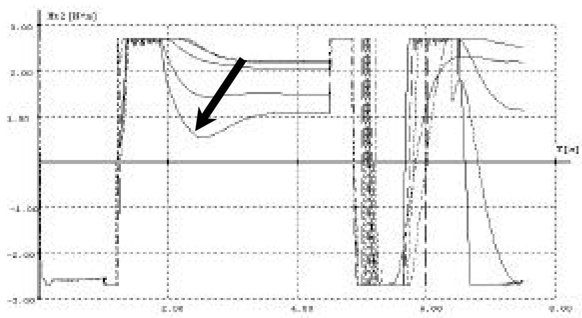
Model of vehicle dynamics

A non-linear multi-body 3D model of 4-wheel passenger car with independent suspension was assumed as the model of vehicle dynamics ([8]). Wheel-road interaction description was based on the extended Dugoff's model (where the transient states of tire according to the von Schlippe-Dietrich relaxation model were accepted). The model of vehicle dynamics had taken also the drive system quasi-dynamics, rolling resistance and aerodynamic drag into account. A simple predictive PD controller was used for programming the steady-state traction conditions.

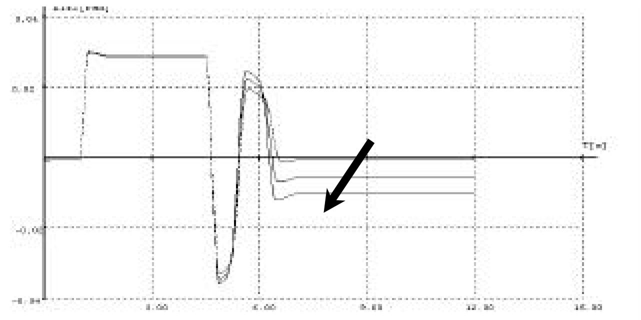
SIMULATION STUDY

These models were used for simulation research (as well as sensitivity and bifurcation analyses) in stick-slip phenomenon in a steering system and vehicle motion. They related to open-loop tests performed according to the respective ISO regulations. Special modular simulation software (together with 3D graphics and animation) has been worked-out.

The paper presents results of simulation research for a car driven at a constant speed of 80 km/h where the following sequence of manoeuvres have been carried out: a step input to the steering wheel, steady state cornering and sudden release of steering wheel. It has been assumed for the basic version that the vehicle was equipped with a manual steering system having a damper in its steering box. For the "richer" versions, power elements of steering system as well as the steering of all four wheels have been also investigated. Exemplary results of simulation presented below refer to varying values of the steering king-pin dry friction magnitude and gear freeplay. They refer to a 2WS manual steering system.



Friction torque at the left king-pin
for increasing gear freeplay



Left wheel angle for
increasing left king-pin dry friction magnitude

The results show a significant influence of both dry friction and freeplay on the stick-slip phenomenon and car's trajectory. The influence of the friction is more visible in steady-states (in the first phase – where the steering wheel has been fixed, and in the second phase – where the turn has been blocked by the stiction). The influence of the freeplay mostly appears in transient states. On the base of investigation, it can be assumed that the friction and freeplay influence on the stick-slip processes and the vehicle motion has been regular. But also some "chaos" referred to levels of steady states was observed for other datasets. On the ground of bifurcation research, it has been ascertained that the freeplay and friction affect the states of blocking in "oscillating fashion" (rises and drops of steady-state values). It could be seen that strong disturbances appear on bifurcation graphs (due to a very large local sensitivity) in certain cases, while in others, horizontal lines occur (due to a local fall in the sensitivity). These have testified to a complicated nature of the modelled processes.

By comparing the results for different structures of the steering system, one could become aware of the fact that the introduction of power steering systems removed the transient processes to a great extent and decreased the sensitivity to the changes of the freeplay and frictions (probably due to increased damping) a little. Similarly, by adding the steering function to the rear wheels the sensitivity has become decreased a little.

CONCLUSIONS

The problems of stick-slip in the steering system lead to more and more complicated (thus, potentially, more and more susceptible to freeplay and friction) mechanisms and control systems in the car.

The method of modelling and simulation of dynamic systems with freeplay and friction taken into account basing on the piecewise linear luz(...) and tar(...) projections appeared very effective.

References

[1] B.Armstrong-Helouvry, P.Dupont, C.Canudas de Wit: A Survey of Models, Analysis Tools and Compensation Methods for the Control of Machines with Friction. *Automatica* **30**, No 7: 1083-1138, 1994.

[2] B.Brogliatto, A.A.T.Dam, L.Paoli, F.Genot, M.Abadie: Numerical simulation of finite dimensional multibody nonsmooth mechanical systems. *Applied Mechanical Review* **55**, No 2: 107-150, 2002.

[3] W.Grzegozek: The Modeling of Vehicle Dynamics with Stabilizing Control of Braking Forces. *Proc. of Cracow University of Technology. Mechanics. Monograph* **275**, 2000 (in Polish).

[4] A.Harlecki: Method of Dynamic Analysis of Multibody Systems with Dry Friction in Joints. *Proc. of Bielsko-Biala University. Monograph*, 2002 (in Polish).

[5] R.A.Ibrahim: Friction-induced vibration, chatter, squeal, and chaos. Part I: Mechanics of contact and friction. Part II: Dynamics and modeling. *Applied Mechanical Review* **47**, No 7: 207-253, 1994.

[6] D.Karnopp: Computer Simulation of Stick-Slip Friction on Mechanical Dynamic Systems. *Transactions of the ASME. J. of Dynamic Systems, Measurement, and Control* **107**: 100-103, 1985.

[7] Z.Lozia: Analysis of Biaxial Car Motion Based upon Dynamic Models. *Proc. of Warsaw University of Technology. Transportation*: **41**, 1998.

[8] Z.Lozia: Vehicle Dynamics and Motion Simulation Versus Experiment. *SAE Paper 980220. SAE Transactions* 1998. *Section 6*, **107**:.344-360, 1998.

[9] Z.Lozia, D.Zardecki: Vehicle Dynamics Simulation with Inclusion of Freeplay and Dry Friction in Steering System. *SAE 2002 Transactions. J. of Passenger Car – Mechanical Systems*: 907-923, 2002.

[10] A.T.Wohnhaas: Simulation von Kraftfahrzeug-Lenkungen unter besonderer Berücksichtigung von Reibung und Spiel. *Dissertation*, IVK Universität Stuttgart, 1994.

[11] D.¯ardecki: Mathematical model of car steering system dynamics with regard to gearbox backlash and friction. *Proc. of Conf. Autoprogres'98*. PIMOT: T.1, 43-52, 1998 (in Polish).

[12] D.Zardecki: The luz(...) and tar(...) projections – theoretical background and idea of application in modelling of discrete mechanical systems with backlashes or frictions. *Biuletyn WAT*: **L**, Nr 5: 125-160, 2001 (in Polish).

[13] D.Zardecki: Simulation Studies of stick-slip process in Car Steering System. Proc. of III Conf. "Problems of Safety in Road Vehicles". Zeszyty Naukowe Politechniki Swietokrzyskiej. Mechanika: 305-312, 2002 (in Polish).