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INERTIAL MIGRATION OF RIGID SPHERICAL PARTICLES IN POISEUILLE FLOW

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Summary A neutrally buoyant particle in pipe flow is known to undergo a radial migration for finite Reynolds number flows. This effect was first observed by Segré and Silberberg (1962), who noted a radial equilibrium position at a radius $r = 0.6R$ in a pipe of radius R, in conditions of finite but low Re. These results have been extended to show that the equilibrium position of the particles is moved toward the wall as Re increases. Moreover a new equilibrium position closer to the center was observed to become dominant at elevated Re. A comparison with asymptotic theories is provided.

In this work, we have examined the influence of inertia upon the radial migration of particles in Poiseuille flow in a tube. We have extended the work of Segré & Silberberg [1] for neutrally buoyant particles up to $Re \approx 2000$. Our results show that the equilibrium radial position for this case, which we have termed the Segré-Silberberg annulus, moves toward the wall as Re is increased. This general trend is illustrated by Figure 1. Also seen on this figure is that our data for large particles, with ratio of tube diameter D to particle diameter d of $D/d < 10$ are in agreement with those of the Segré & Silberberg experiments performed for particles at similar D/d. The results for small particles, $D/d = 42$, are in good agreement with predictions of asymptotic theory [2–5] up to $Re \approx 500$. Note that the results for large particles do not agree with the asymptotic theory: in particular, the experimentally observed equilibrium position is moved toward the center of the pipe relative to the predictions of this theory. This deviation from theory increases as D/d becomes smaller (increasing particle size for fixed tube diameter). The discrepancy can be explained as a particle size effect, as the theory is strictly valid only for $Re_p \ll 1$, a condition which is not met in our experiments. In addition, the theory was developed for a channel flow, and the curvature of the pipe in the experiments cannot be neglected for small D/d. A further source of discrepancy is that the theory may break down for an equilibrium position too close to the wall.

In these experiments, we have observed an accumulation of particles on an inner annulus, i.e. at smaller radial position than the Segré-Silberberg annulus. This occurs for $Re > 600$ for all particles except the smallest set studied here of $D/d = 42$, for which this inner annulus was observed only for $Re > 1200$. This inner annulus coincides with the change in concavity of the force profile predicted by asymptotic theory for channel flow at large Re. In this work, we have probed whether this inner annulus corresponds to a true zero of the force or is a transient feature. Simulations of the trajectories of particles using the force deduced from the asymptotic theory show an inner annulus as a transient feature, meaning particles are seen on the inner annulus at intermediate axial locations before completion of migration to the Segré-Silberberg annulus further downstream. However, this inner annulus is very robust at large Re in the sense that it is even observed to survive through the transition to intermittency, despite the fact that the entry length becomes much smaller than the pipe length at these large values of Re according to existing theory. Also, the distribution is found to change from one centered at the Segré-Silberberg annulus to one with the particles primarily on the inner annulus when Re is increased. This change is seen between in Figure 2, where the distribution at the measurement cross-section and the histogram showing probability of lying at a dimensionless radius are shown for $Re = 1000$, 1650, and 2400 (the last is above transition to intermittent turbulence). In the results from $Re = 1000$ note that the particles are divided between the inner and Segré-Silberberg annuli, while at the larger Re only the inner annulus is populated. This transition depends upon Re in the same way for all particles except the smallest set, a finding in complete contradiction with the theoretical prediction of entry length varying as $(D/d)^3$. These findings lead us to believe that the inner annulus is not a transient feature but is a genuine equilibrium position not predicted by the asymptotic theory for Poiseuille flow in a channel. As noted above, this theory lacks two key features, the curvature of the pipe and the finite size of the particles, which if included may provide insight into the appearance of the inner annulus.

We have investigated the influence of buoyancy in two cases, one being a slight dispersion of particle density about that of the fluid, and the second for a set of particles all slightly heavier than the fluid. For the case of a dispersion in density, we observe an accumulation at the top and bottom of the cross section. This accumulation disappears as the ratio of inertia to buoyancy is increased through an increase of the Reynolds number. In the second case, the inertial migration competes with settling above the tube centerline while the two act in concert below, and this results in a strong asymmetry of the distribution. In both cases, simulation of the particle trajectories based on asymptotic theory captures the general features observed in the experiments.

References

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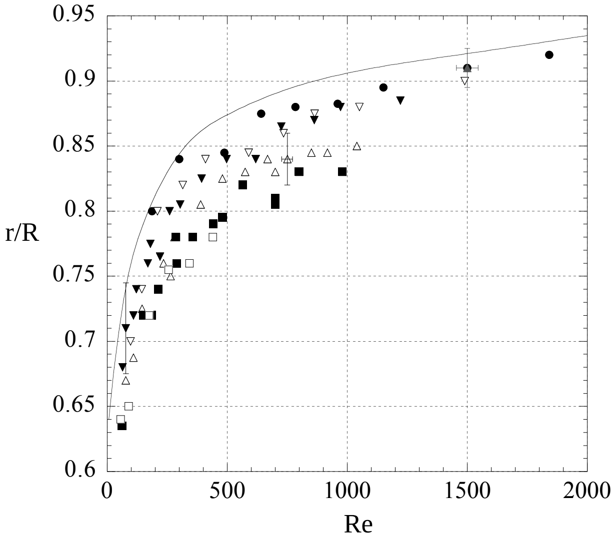

Figure 1. Segré-Silberberg equilibrium position of a rigid spherical particle as a function of Reynolds number for different particle sizes: ● $D/d = 42$, ▽ $D/d = 17$, ▼ $D/d = 15$, △ $D/d = 10.5$, ■ $D/d = 9$, □ $D/d = 8$. The solid curve represents the prediction of the asymptotic theory.

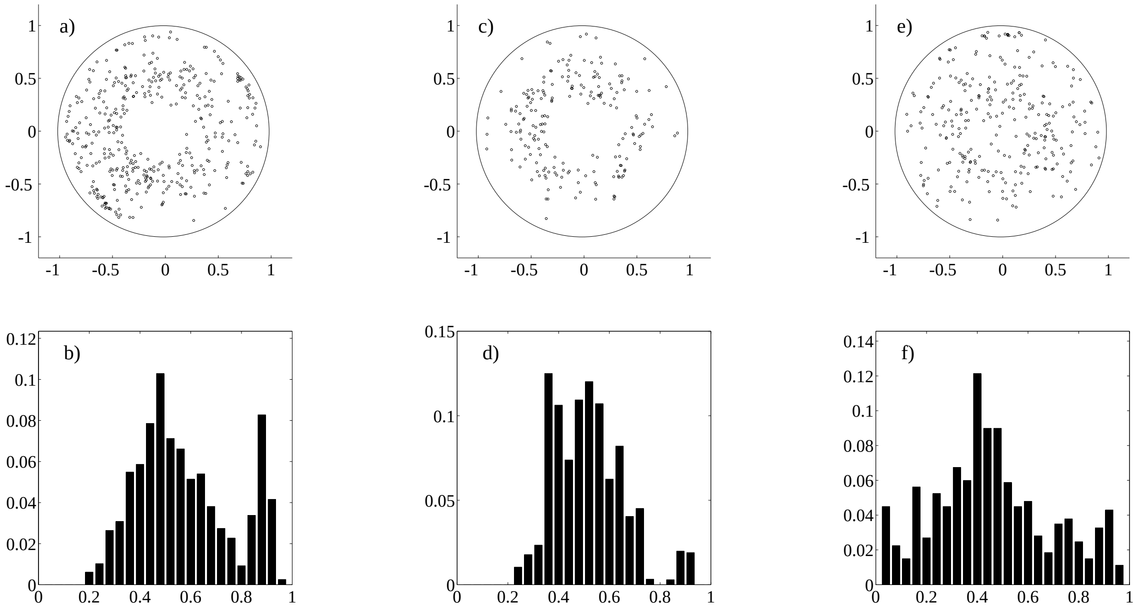

Figure 2. Particle distributions over a cross section for $D/d = 17$ at a) $Re = 1000$, c) $Re = 1650$ and e) $Re = 2400$ and the respective histograms b), d), and f) showing the probability of finding a particle at a given dimernsionless radius, $p(r)$. In a), c), and e), both axes are labeled with lengths scaled by the tube radius. Note that at $Re = 2400$ the flow is intermittent.