ON MODELING THE LONGITUDINAL IMPACT OF TWO SHAPE MEMORY BARS

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Summary We study the propagation of phase transformation fronts induced by the longitudinal impact of two shape memory bars. The corresponding wave structure is investigated by using the non-monotone elastic model versus a Maxwell's rate-type model containing a rate sensitivity parameter and a time of relaxation of kinetic origin. Laboratory measurements are suggested.

Experiments with longitudinal impact of SMA bars are an effective mean for understanding the kinetics of phase transformation. We investigate here this problem in an one-dimensional and isothermal setting since this study provides an important insight into the wave structure. In a future work we extend the analysis to account for thermal effects.
The balance laws of momentum and mass in a 1-D Lagrangian description are

$$\varrho\frac{\partial v}{\partial t} - \frac{\partial \sigma}{\partial X} = 0, \qquad \frac{\partial \varepsilon}{\partial t} - \frac{\partial v}{\partial X} = 0, \tag{1}$$

where v, σ, ε and ϱ are the particle velocity, the stress, the strain and mass density, respectively.
The theory of elasticity with non-monotonic stress-strain relation $\sigma = \sigma_{eq}(\varepsilon)$, has been frequently used to describe the main features of phase transformations in solids. For a three-phase elastic material the simplest non-monotonic stress-strain relation is given in Fig.1. This constitutive relation is viewed as corresponding to a material which can exist in an *austenite phase* and in two *variants of martensite*. E_1 =const.>0 is called the elastic modulus of the austenite phase, while E_3 =const.>0 is the elastic modulus of the martensite variants. $-E_2$ =const.<0 is called the softening modulus and it corresponds to the unstable phases of the material.

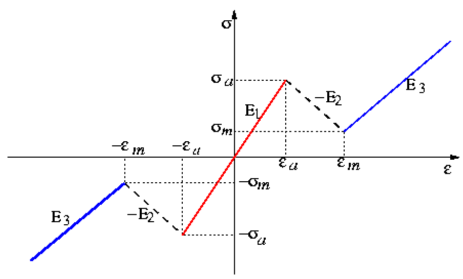

Figure 1: The three phase elastic material - equilibrium curve $\sigma = \sigma_{eq}(\varepsilon)$.

It is known that initial-boundary value problems for the non-monotone elastic system (1)+($\sigma = \sigma_{eq}(\varepsilon)$) can be ill-posed. This situation reflects in fact a constitutive insufficiency of the model. One way to remedy it is to embed the elastic bar theory as a special case of a broader theory which can describe the way the body may deviate from the equilibrium. We adopt in this paper the following Maxwellian rate-type constitutive equation

$$\frac{\partial \sigma}{\partial t} - E\frac{\partial \varepsilon}{\partial t} = -\frac{E}{\mu}|\sigma - \sigma_{eq}(\varepsilon)|^{\lambda-1}(\sigma - \sigma_{eq}(\varepsilon)), \tag{2}$$

where $\sigma_{eq}(\varepsilon)$ is a non-monotone stress-strain relation as in Fig.1 . E =const.>0 is the *dynamic Young modulus*. μ =const.>0 is a *Newtonian viscosity coefficient*, $\frac{\mu}{E}$ is a *relaxation time* of the model. λ =const.>0 is a *rate sensitivity parameter*. When $\mu \to 0$ this rate-type constitutive equation can be seen as a viscoelastic approach of the elastic model in a $L^{\lambda+1}$ sense.
From mathematical point of view, the rate-type system (1)+(2) has the advantage that it is always *hyperbolic semilinear*. The initial-boundary value problems are now well-posed even in the unstable phases. It was shown in [3], for the case $\lambda = 1$, that this system incorporates some physical instabilities due to the shape of the equilibrium curve and it is able to describe general issues pertaining to phase transition phenomena. The rate-type constitutive relation (2) has the capacity to describe the nucleation of phases. The new parameters μ, λ and E influence in fact the kinetics of the growth of phases and should be connected with the time of growth or time of nucleation of microscopic theories of phase transitions.
This work is organized as follows. First we investigate analytically the wave structure corresponding to the rate-type system (1)+(2). By following [1], we analyze the behavior of the waves near the source and far from

the source of a perturbation. Thus we determine which set of waves are the most important and will be really observed. By using a method of multiple scales we establish that near the source the waves propagate with the speeds $\pm\sqrt{E/\varrho}$ (*instantaneous waves*) being immediately exponentially damped. On the other hand, in the unstable phases any inhomogeneity in the strain field leads to the nucleation and, consequently to the formation of a *stationary phase boundary.* Far from the source, the main part of the disturbance moves with the speeds $\pm\sqrt{E_0/\varrho}$ (*delayed waves*), where E_0 is the elastic modulus of the austenite phase (E_1) or of the martensite variants (E_3). The disturbance decreases like $1/\sqrt{at}$, which is typical for diffusion and spreads out like $\sqrt{at}$ where $a = 2(E - E_0)\mu/\varrho E$. In order to determine the speed of propagation of phase boundaries we investigate *traveling wave solutions* for the Maxwell's type model. We establish an admissibility criterion to identify those moving phase boundaries within the elastic theory. This is just the well known *chord criterion.* The viscosity parameter μ, the rate sensitivity parameter λ and the dynamic Young modulus E will only influence the smooth transition layer which approximate a sharp interface.

The main goal of this work is to investigate the longitudinal impact of two phase transforming bars. Thus we consider at the initial moment a bar called "target" impacted at one end by another bar called "flyer" which is moving with a known initial velocity V. After impact the two bars remain in contact and move together until a time t_S called *time of separation.* This time corresponds to the moment when the first tensile wave arrives at the point of contact $X = 0$.

Exact solution - First we construct an exact solution of this problem using the elastic model and the chord criterion. The building blocks in solving this problem are the simplest initial-boundary value problems for the elastic system, i.e., the Goursat and Riemann problems which are thoroughly investigated. If the impact velocity V is larger than $V_{ph} \equiv 2\sigma_a/\sqrt{\varrho E_1}$ then a phase transformation is induced. Interactions of the unloading elastic wave reflected at the free end of the target with the propagating phase boundary are explored. Critical values of the impact velocity are determined such that this phase boundary propagates backward, remains stationary or propagates forward.

Numerical solution - We determine numerically the evolution in time of the stress, strain and particle velocity in the " flyer" and "target" bar predicted by the rate-type system (see Fig. 2). Increasing the impact velocity, the transforming region increases and the separation time changes. Inside the transforming region longitudinal waves propagate reflecting at the phase boundaries and also being transmitted across them. Velocity time profiles obtained at the rear end of the target provides additional information for understanding the forward and reverse transformation kinetics. We focus on the numerical results which can be compared with experimental data: the time of separation (optical methods), the velocity at the end of the target bar (interferometry), the stress at the impacted end (piezoelectric wafers), the variation in time of the strain at various cross-sections (diffraction gratings). Some connections with experimental results in which the transformation was obtained by pressure-shear plate experiments (see [2]) are discussed.

The numerical experiments for the Maxwell's rate-type system illustrate the potentiality of the adopted model to describe specific phenomena accompanying stress-induced phase transformations during impact tests.

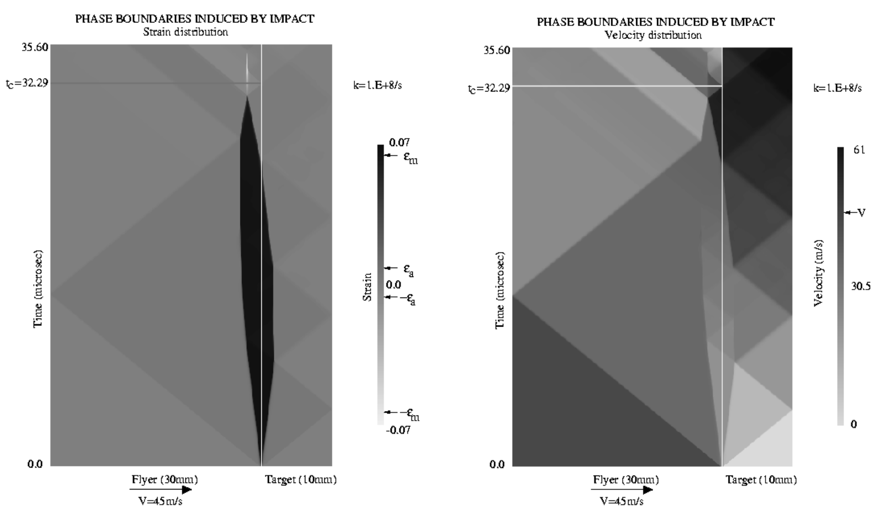

Figure 2:

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