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THERMO-MECHANICAL WAVE PROPAGATION IN MATERIALS WITH INTERNAL STATE VARIABLES

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Summary Conditions imposed on the initial data by the assumption that an acceleration wave may propagate in a termo-visco-plastic material and its amplitude blow-up to infinity in finite time are discussed. Full termomechanical coupling in 3D case is considered. Constitutive model is derived from a free energy function taking into account the non-negativity of entropy production. Thermal properties are characterized by the dependence of the heat flux on the gradient of a new thermal variable called the semi–empirical temperature scale. The theory leads to a modified model of thermo-elasto-viscoplasticity with an extra thermal stress effect and wave-type heat conduction. In the case of plane waves the solution is constructed numerically even beyond the point where Lipschitz continuity is lost and a shock with finite amplitude arises. Conditions under which a pure mechanical disturbance generates thermal one and the both may lead to blow up are established. Such a situation can model the ultimate state of a viscoplastic material in the vicinity of the dynamically propagating crac tip.

We study the propagation of thermo-mechanical waves in a nonlinear material with internal state variables. The general framework of such material models was proposed by Valanis[9], Coleman and Gurtin [1]. It was further developed by Perzyna, and Kosiński [6], who were the first to study wave propagation in this context. The results of those derivations were applied to 3D thermoelasticity in [8]. Further generalization for more complex materials was done in [10]) and then partially reported in the monograph [5]. The theory of thermal waves at low temperatures was developed in co-operation with V. A. Cimmelli and K. Frischmuth [2]. For this case material functions were identified and numerical solutions obtained. In co-operation with K. Saxton analytical results on weak singularities (acceleration waves) were developed, ordinary differential equations for the amplitude of acceleration waves were discussed and the possibility of blow up shown [7]. Next, the developed thermal models were imbedded into models of deformable bodies, first thermoelastic, then thermo-visocoplastic ones, [4, 3].

The presented constitutive models were derived from a free energy function taking into account the non-negativity of entropy production. In this way a representation of material functions is facilitated even in a full 3D setting. In a recent paper, [3], the mechanical part is governed by Perzyna's *overstress* function. The thermal properties are characterized by the dependence of the heat flux on the gradient of a new thermal variable called the *semi–empirical temperature scale* as studied before in the case of rigid and elastic heat conductors.

The theory leads to a modified model of thermo-elasto-viscoplasticity with an *extra thermal stress effect* and wave–type heat conduction. This contribution can have a substantial meaning in describing thermo-mechanical coupling phenomena in viscoplastic materials, especially in the context of fracture and damage phenomena under very high strain rates [11]. Moreover, even for isotropic material extra thermal stress introduces anisotropy due to its dependence on the heat flux vector.

When introduced into the general system of balance equations for mass, momentum and energy, the material functions and kinetic equations lead to a system of quasi–linear hyperbolic equations.

For such systems acceleration waves as well as shock waves and rarefaction fans are of interest. Following our previous papers[4, 3], the coupled system of field and constitutive equations modelling 3D thermo-mechanics at finite inelastic strain is derived and analyzed in the case of solutions with weak discontinuities.

Let us notice that in 3D case we have a system of four homogeneous equations for two amplitudes: the vector–valued mechanical amplitude $\mathbf{s}$ and the scalar thermal one v. To get a nontrivial solution, the determinant of the system has to vanish. This gives us the so-called characteristic (or dispersion) relation for the normal speed U of the wave front in terms of the unit normal to the wave front $\mathbf{n}$ and values of state variables at the front. In general thermomechanical wave propagation is not symmetric like in the other models, cf.[6, 8, 10]. Amplitudes of those discontinuities satisfy ordinary differential equations of Bernoulli type, [7].

In the present paper, we discuss the conditions imposed on the initial data by the assumption that an acceleration wave may propagate and its amplitude blow-up to infinity in finite time. Full termomechanical coupling is considered; which leads to non-zero amplitudes of the both types. Furthermore, we analyze such situations numerically. In the case of plane waves this enables us to construct the solution approximately even beyond the point where Lipschitz continuity is lost and a shock with finite amplitude arises. Conditions under which a pure mechanical disturbance generates thermal one and the both may lead to blow up are established. Such a situation can model the ultimate state of a viscoplastic material in the vicinity of the dynamically propagating crac tip [11].

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