OPTIMAL DESIGN OF UNCONSTRAINED DAMPING LAYER ON BEAMS

Doo-Ho Lee*, Woo-Seok Hwang**
*Department of Mechanical Engineering, Dongeui University, 614-714, Busan, Korea
**School of Automotive/Industrial and Mechanical Engineering, Daegu University, Korea

Summary The optimum layout of unconstrained damping layer on beams is obtained using an equivalent stiffness approach and finite element formulation. The Ross, Ungar and Kerwin(RUK)'s formula is introduced to represent the equivalent complex modulus of the damping layer and beam. The fractional derivative model describes the dynamic characteristics of viscoelastic materials in order to include the non-linearities of real materials with respect to frequency and temperature. Using the equivalent stiffness, a finite beam element is developed and a nonlinear eigenvalue problem is solved for a beam with the unconstrained damping layer on it. The objective of optimization problem is to maximize the product of loss factor and eigenfrequency of a specified mode. Optimum coverage is obtained by combining an analytic design sensitivity analysis and a gradient-based numerical search algorithm.

INTRODUCTION

The damping of structures is one of the most important factors in order to reduce the vibrations of structures. Authors suggested a layout optimization formulation of unconstrained damping layer in Ref [1] that gives maximum loss factor for a specified mode. However, it is noticed that the resonance frequency at the optimal damping layout could be fallen considerably due to mass effect of the damping materials. In this study, optimum layout of unconstrained damping layer on beams that maximizes both the loss factor and the eigenfrequency of a vibration mode will be identified using a numerical search method.

ANALYSIS OF UNCONSTRAINED VISCOELASTIC DAMPING LAYER

Dynamic characteristics of the viscoelastic materials in frequency domain can be represented using the complex modulus such as $\bar{\sigma} = E^*\bar{\varepsilon} = E(1+\boldsymbol{i}\eta)\bar{\varepsilon}$ where $\boldsymbol{i} = \sqrt{-1}$, and $\bar{\sigma}$ and $\bar{\varepsilon}$ are the Fourier transforms of stress and strain, respectively. E^*, E and η are the complex modulus, the storage modulus and the loss factor, respectively. Many environmental factors affect to the dynamic characteristics of the viscoelastic materials. Particularly, the complex modulus of the viscoelastic material is strongly dependent on temperature and frequency. In order to consider the effects of temperature on damping behavior, there is a well-known temperature-frequency superposition principle that says the temperature effects can be converted to those of frequency using a shift factor, $\alpha(T)$ where T is temperature. The shift factor and temperature can be related by the Arrhenius equation such as $log(\alpha(T)) = d_1(1/T - 1/T_0)$. The fractional derivative model represents the damping elements as a time derivative of order smaller than unity. The constitutive equation of the fractional derivative model of order one can be written as:

$$\sigma(t) + c_1 D^\beta \sigma(t) = a_0\varepsilon(t) + a_1 D^\beta \varepsilon(t) \quad \text{where} \quad D^\beta\sigma(t) = \frac{1}{\Gamma(1-\beta)}\frac{d}{dt}\int_0^t \frac{\sigma(\tau)}{(t-\tau)^\beta}d\tau \tag{1}$$

and $0 < \beta < 1$, and a_0, a_1, c_1 and β are material parameters. Then, the complex modulus of viscoelastic materials obtained by the Fourier transforms of equations (1) are expressed as:

$$E^* = E(1+\boldsymbol{i}\eta) = \{a_0 + a_1(\boldsymbol{i}2\pi f\alpha(T))^\beta\}/\{1 + c_1(\boldsymbol{i}2\pi f\alpha(T))^\beta\} \tag{2}$$

Consider an unconstrained damping layer beam as shown in Fig. 1. The storage modulus and the loss factor of the viscoelastic damping layer are E_2 and η_2, respectively. The storage modulus, the loss factor and the second area moment of the base beam are E_1, η_1 and I_1, respectively. The equivalent complex flexural rigidity, E^*I, of the unconstrained beams by RUK's equivalent rigidity method is written in the form:

$$E^*I / E_1^*I_1 = 1 + e^*h^3 + 3(1+h)^3 e^*h/(1+e^*h) \tag{3}$$

where $h = H_2/H_1$, and $e^* = E_2^*/E_1^*$. The unconstrained damping layer beams can be modeled using a finite beam element formulation with the equivalent flexural rigidity. Following standard finite element formulation procedure with the equivalent flexural rigidity, one can define the corresponding eigenvalue problem as follows.

$$[\boldsymbol{K}]\{\boldsymbol{y}\} = \varsigma[\boldsymbol{M}]\{\boldsymbol{y}\} \tag{4}$$

where $[\boldsymbol{K}] = [\boldsymbol{K}_{Re}] + \boldsymbol{i}[\boldsymbol{K}_{Im}] = [\boldsymbol{K}_{Re}](1+\boldsymbol{i}\eta)$ and $[\boldsymbol{M}]$ and $[\boldsymbol{K}]$ are the global mass and stiffness matrices, respectively. Subscripts *Re* and *Im* mean the real and imaginary parts, respectively, and, the vector, $\{\boldsymbol{y}\}$, is the eigenvector and $\varsigma(=\omega^2 = (2\pi f)^2)$ is the eigenvalue. The eigenvalue

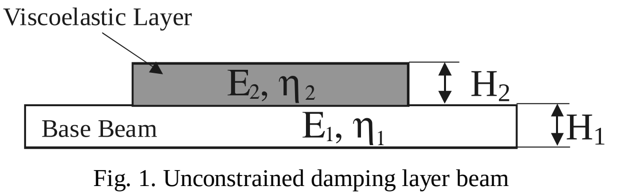

Fig. 1. Unconstrained damping layer beam

problem of Eq. (4) is nonlinear equations because the stiffness matrix is a function of frequency due to the viscoelastic damping layer. In order to solve the nonlinear eigenvalue problem of Eq. (4), an iteration procedure is necessary[1]. The loss factor of a structure for a vibration mode is defined as:

$$\eta^k = \sum_{j=1}^{p} \eta_j U_{ej} \Big/ \sum_{j=1}^{p} U_{ej} = \sum_{j=1}^{p} \eta_j U_{ej} \Big/ U \qquad (5)$$

where η^k is the loss factor of the k-th mode, p is the number of finite elements, η_j is a loss factor of the j-th element, and U_{ej} is the strain energy of the j-th finite element.

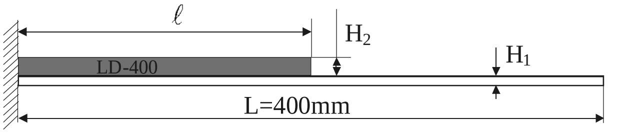

Fig. 2. A clamped-free beam

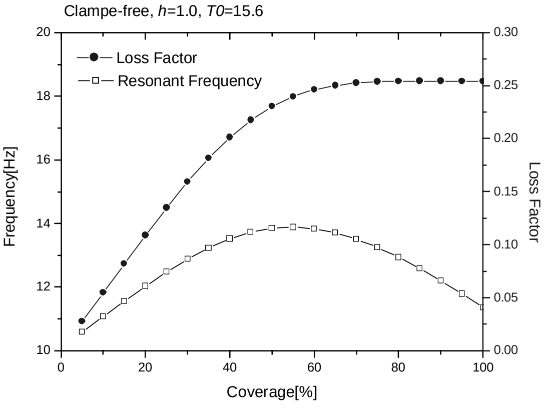

Fig. 3. Variation of loss factor and resonant frequency with respect to coverage of damping layer (h=1)

SENSITIVITY ANALYSIS AND LAYOUT OPTIMIZATION OF DAMPING LAYER

Assuming a uniformly-coated damping layer for a practical consideration, the optimal design problem of the unconstrained damping layout for a specified k-th mode on beams can be defined as follows.

$$\begin{aligned}&\textit{Find the design variables } \mathbf{b} \textit{ such that}\\ &\textit{maximize } \ \eta^k(\mathbf{b}; f,T)\times\varsigma^k \ \textit{ subject to } \mathbf{b}_L \le \mathbf{b} \le \mathbf{b}_U\end{aligned} \qquad (6)$$

where $\mathbf{b}$ is the design variables and $\mathbf{b}_L$ and $\mathbf{b}_U$ are the lower and upper bounds of the design variables. Design sensitivity of the loss factor can be obtained from Eqs. (2), (3) and (5) using the chain rule as follows.

$$d\eta^k/db = \{ \sum_{i=1}^{m} \{d\eta_i(b,f,T)/db \cdot U_{ei} + \eta_i \cdot dU_{ei}/db\} - \eta^k \cdot dU/db \}/U \qquad (7)$$

$$d\eta_i(b,f)/db = \partial\eta_i/\partial b + \partial\eta_i/\partial f \cdot df/db \ \text{ where } \ \partial\eta_i/\partial f = (Im(\partial \mathbf{E}^*/\partial f)\cdot Re(\mathbf{E}^*) - Im(\mathbf{E}^*)\cdot Re(\partial \mathbf{E}^*/\partial f))/(Re(\mathbf{E}^*))^2 \qquad (8)$$

Eqs. (7) and (8) can be calculated using the following i-th eigenvalue and eigenvector sensitivity formulae.

$$\dot{\varsigma}^i = \{\mathbf{y}^i\}^T[\partial \mathbf{K}/\partial b]\{\mathbf{y}^i\} - \varsigma^i\{\mathbf{y}^i\}^T[\partial \mathbf{M}/\partial b]\{\mathbf{y}^i\} \qquad (9)$$

$$\{\partial \mathbf{y}^i/\partial b\} = \sum_{j=1, j\neq i}^{r} -\{\mathbf{y}^j\}^T([\partial \mathbf{K}/\partial b] - \varsigma^i[\partial \mathbf{M}/\partial b])\{\mathbf{y}^i\}/(\varsigma^j - \varsigma^i)\{\mathbf{y}^j\} - 0.5(\{\mathbf{y}^i\}^T[\partial \mathbf{M}/\partial b]\{\mathbf{y}^i\})\{\mathbf{y}^i\} \qquad (10)$$

Therefore, in order to obtain the loss factor sensitivity information, it is necessary to solve one eigenvalue problem, one eigenvalue sensitivity analysis and one eigenvector sensitivity analysis. To validate the optimization formulation of the unconstrained damping layer beams, a numerical example is introduced as shown in Fig. 2. In Fig. 2 a viscoelastic damping material, LD-400[2], is bonded on an aluminum beam. The damping-treated beam is modeled with 20 beam elements. Fig. 3 shows variation of the loss factor and the first resonant frequency. As shown in Fig. 3, the resonant frequency goes down before the loss factor reaches to the maximum. Optimal lengths of the unconstrained damping layer that gives maximum product of the loss factor and eigenfrequency for the first mode are determined using a gradient-based search algorithm. The design variable is the length of the damping layer. Fig. 4 shows the optimal coverage does not vary considerably contrary to the case of maximum loss factor only in Ref. [1].

CONCLUSIONS

Optimal damping treatment layouts on the unconstrained beams are identified according to the thickness ratios and the environmental temperatures. The optimization results show that the proposed formulation is effective for real viscoelastic damping materials.

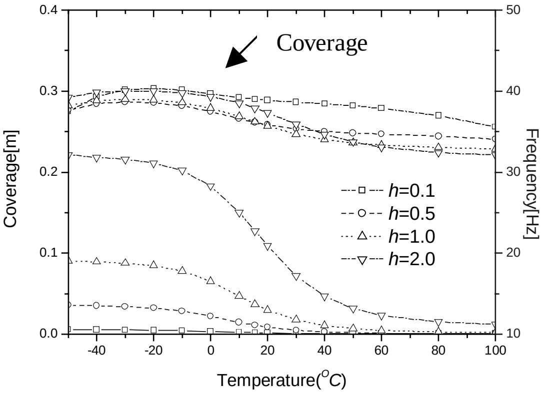

Fig. 4. The optimum coverage of damping layer and the corresponding resonant frequency for the first mode

References

[1] Lee D.-H., Hwang W.-S.: Layout Optimization of Unconstrained Viscoelastic Layer on Beams Using Fractional Derivative Model, *AIAA J.(Submitted for publication)*, 2004.

[2] Jones D.G: Handbook of Viscoelastic Vibration Damping. John Willy & Sons, NY 2001.