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JUMPING OF A SPINNING SPHEROID

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Summary As is well known, a hard-boiled egg will rise from the horizontal to the vertical if it is spun sufficiently rapidly on a table with its axis of symmetry initially horizontal. We consider here the problem of a spinning spheroid, and show that in certain circumstances the spheroid may lose contact with the table in the course of this rising motion. Allowing for slip and weak friction at the point of contact, the dynamical equations for a uniform spheroid, are treated by the multiple-scale perturbation method to resolve the two time-scales intrinsic to the dynamics. An approximate solution for the high frequency component of the motion shows a growing oscillation of the normal reaction, and predicts the circumstances in which this can fall to zero (leading to jumping of the body). The exact solution for the free motion after jumping and until contact with the table is reestablished. The analytical results agree well with numerical simulations of the exact equations.

BACKGROUND AND OBJECTIVE

Moffatt & Shimomura [1] discussed the familiar phenomenon of the rise to the vertical of a hard-boiled egg set in rapid spinning motion on a table. In that paper, the governing equations were simplified on the assumption that the friction is weak and the spin is large (so that the Coriolis force is dominant). Under this 'gyroscopic' assumption, a first-order differential equation for the inclination of the axis of symmetry was obtained, which, for the case of a prolate spheroid, did indeed describe the rise of the axis to the vertical. This rise was associated with the effect of the weak friction at the point of contact, and occurred on a 'slow' time-scale $O(\mu^{-1})$, irrespective of the nature of the frictional force ('dry' Coulomb friction, or 'wet' viscous friction). However this rising motion is accompanied by fluctuations occurring on a 'fast' $O(1)$ time-scale. In fact, a carbon trace of the moving point of contact, which is initially a continuous curve, can turn into a broken and then a dotted one in an intermediate stage of the rising motion (See Figure 1). This at least means a rapidly oscillating fluctuation of the normal reaction R acting on the point of contact. Furthermore, a linear stability analysis by Moffatt, Shimomura & Branicki [2] showed the existence of modes oscillating on 'fast' time-scale. These 'slow' and 'fast' time-scales embedded in the dynamics are expected to be clearly separated when the friction is weak. In the papers [1] and [2], the computations presented were restricted to circumstances in which R remains positive and the spheroid does therefore remain in contact with the table throughout the motion. However, in some regions of parameter space, the growth of the fluctuations allows R to decrease to zero. The purpose of the present paper is to analyse these fluctuating motions, which depend on the two time-scales, and to find their effect on the dynamics.

METHODS

The motion of a spinning spheroid occuring on slow and fast time-scales is analytically studied for small Coulomb friction parameter μ, using the multiple-scale perturbation method. First, two time-scales are introduced and the mean part of the motion is defined by filtering out the fast motion. (In reference [1], the solutions for this mean part was found under the 'gyroscopic' assumption.) The fluctuating parts to order μ are approximately solved by the WKB method to derive the time-dependence of the normal reaction R. Numerical simulation of the problem is also carried out by using the Runge-Kutta method to solve the exact governing sixth-order nonlinear dynamical system.

RESULTS

The following results are found analytically and confirmed numerically. The normal reaction R for a spinning spheroid oscillates with a growing amplitude. Depending on the aspect ratio and the initial condition, the oscillatory modes can grow to such an extent that R can fall to zero (see Figure 2). This leads to a 'jumping' phenomenon, in which obviously the spheroid passes into a free motion under gravity without friction. The exact solution for this free motion describes the distance of the gap between the spinning spheroid and the table, which increases in proportion to t^3 in the initial stage of jumping. The time of free motion before contact is determined.

CONCLUSION

If a spinning spheroid is spun sufficiently rapidly on a table with its axis of symmetry horizontal, then it will lose contact with the table at some stage during the rising motion [3]. In these circumstances, its subsequent behaviour consists of rapidly alternating periods of motion with and without frictional contact with the table.

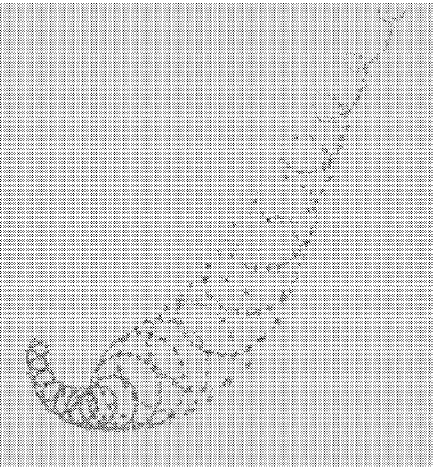

Figure 1. Carbon trace of the point of contact for a spinning prolate spheroid of aspect ratio 3/2.

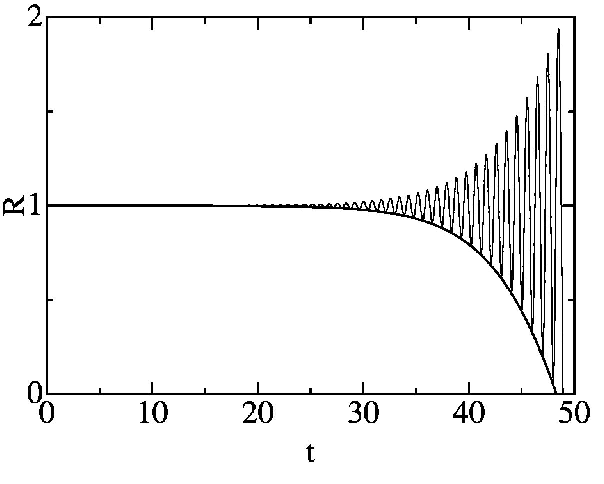

Figure 2. Normal reaction force R versus time t for a spinning prolate spheroid in the case of "Coulomb" friction. The thin line shows the numerical simulation, and the bold one plots the theoretical result.

References

[1] Moffatt, H. K. & Shimomura, Y.: Spinning Eggs – a Paradox Resolved. Nature 417:385–386, 2002.

[2] Moffatt H. K., Shimomura, Y. & Branicki, M.: Dynamics of an Axisymmetric Body Spinning on a Horizontal Surface. Part I: Stability and the Gyroscopic Approximation. to appear in Proc. Roy. Soc. A, 2004.

[3] Shimomura, Y., Branicki, M. & Moffatt H. K.: Dynamics of an Axisymmetric Body Spinning on a Horizontal Surface. Part II: Self-Induced Jumping. in preparation.