STABILITY OF A SPINNING DISK UNDER A STATIONARY OSCILLATING UNIT

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Summary This work studies the free vibration and stability of a spinning disk transversely in contact with a stationary oscillating unit. The oscillating unit consists of two parallel combinations of springs and dampers attached above and under a mass such that the displacement of the mass is different from that of the disk at the contact point. The stability analysis is conducted by investigating the eigenvalue problem of the combined system.

Spinning disks are widely used components in mechanical engineering, from circular saw blades, turbine rotors to computer disk memory units. Therefore, the dynamic behaviors of spinning disks have attracted researchers' interest for a long time. For computer disk memory units, the interaction of the read/write head with the surface of the disk demands that the effects of the inertia, stiffness and damping of the head be considered in the analysis. Hence, Iwan and stahl [1] first studied the free vibration and stability of a stationary circular disk excited by a rotating mass-spring-damper load system. The displacement of the mass of the load system is assumed to be equal to the transverse deflection of the disk. Later Iwan and Moeller [2] included the rotational inertia effect of the disk and investigated the free vibration and stability of a spinning disk with a stationary mass-spring-damper load system. They found that the primary effect of the disk rotation was to stiffen the disk and thereby to increase effective natural frequencies over those of a stationary disk with a rotating load system.

In reality, the displacement of the mass of the load system is generally not the same as that of the disk at the contact point. Consider an annular plate that is spinning at an angular speed Ω and is transversely in contact with a stationary oscillating unit at the point $(r_P, 0)$, where (r,θ) is a polar coordinate system fixed in space. The plate is clamped at the inner edge and free at the outer edge, and the oscillating unit consists of two parallel combinations of springs and dampers attached above and under a mass. The lower end of the oscillating unit is assumed to contact with the disk closely, and the upper end of it is fastened to a fixed support. With respect to the inertial polar coordinates, the equation of motion of a spinning disk with viscous damping under the action of the stationary oscillating unit can be written as

$$D\nabla^4 w + c\left(\frac{\partial w}{\partial t} + \Omega\frac{\partial w}{\partial \theta}\right) + \rho h\left(\frac{\partial^2 w}{\partial t^2} + 2\Omega\frac{\partial^2 w}{\partial t\partial\theta} + \Omega^2\frac{\partial^2 w}{\partial \theta^2}\right)$$

$$= h\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\sigma_{rr}\frac{\partial w}{\partial r}\right) + \frac{1}{r^2}\sigma_{\theta\theta}\frac{\partial^2 w}{\partial\theta^2}\right] + \frac{1}{r_P}(c_P\dot{y} + k_P y)\delta(r - r_P)\delta(\theta) \tag{1}$$

where D and c are the flexural rigidity and the viscous damping coefficient of the disk, respectively; ρ and h are the mass density and the thickness of the disk, respectively. σ_{rr} and $\sigma_{\theta\theta}$ are the initial in-plane stresses induced by rotation. c_P and k_P are the damping coefficient and the spring constant of the lower damper and spring of the oscillating unit, respectively. ∇^4 is a biharmonic operator, and $\delta(\cdot)$ is a Dirac delta function. w is the transverse displacement of the disk. y is the relative displacement of the mass of the oscillating unit, and an overdot denotes a differentiation with respect to time t. The equation of motion of the oscillating unit is given by

$$m_o \ddot{y} + (c_S + c_P)\dot{y} + (k_S + k_P)y = -m_o \frac{\partial^2 w}{\partial t^2}(r_P, 0, t) - c_S \frac{\partial w}{\partial t}(r_P, 0, t) - k_S w(r_P, 0, t) \tag{2}$$

where c_S and k_S are the damping coefficient and the spring constant of the upper damper and spring of the oscillating unit, respectively. m_o is the mass of the oscillating unit.

Assume that the displacement of the spinning disk can be expressed as a linear combination of the eigenfunctions of the corresponding stationary disk. Substituting the assumed displacement into Eq. (1), going through Galerkin's procedure and combining Eq. (2) yields the discretized system equations for the combined system - the spinning disk and the oscillating unit. The discretized system equations of the combined system can be rewritten into a set of the first-order differential equations, and the solution of this set of differential equations has an exponential form. Substituting the solution into the differential equations yields an eigenvalue problem. The eigenvalues and the eigenvectors of the eigenvalue problem appear in complex conjugate pairs. When the real part of an eigenvalue becomes positive, the corresponding mode is unstable. Furthermore, if the imaginary part of this eigenvalue is equal to 0, the corresponding mode experiences a divergence-type instability; if the imaginary part of this eigenvalue is not 0, the corresponding mode experiences a flutter-type instability.

If the inner radius of the disk is assumed to approach zero, and the spring constant and viscous damping coefficient of the lower spring and damper are assumed to approach infinity such that the displacement of the mass is the same as that of the disk at the contact point, the problem considered in this work is reduced to that studied by Iwan and Moeller [2]. The figure of the eigenvalues versus the spin rate reveals that the locations and the widths of the unstable intervals obtained in this study agree excellently with those in Iwan and Moeller's paper. By piling up the unstable intervals with respect to a certain parameter, stability boundaries of the combined system can be obtained. Parametric studies of the system parameters on the stability boundaries of the combined system are conducted numerically.

Numerical results show that inclusion of the spring between the disk and the mass of the oscillating unit will bring about new and larger unstable regions between the oscillating unit and the reflected modes of the disk but repress the original instability by the reflected and backward modes of the disk, of which the unstable regions are much smaller. An increase in the spring constant of either the lower spring or the upper spring of the oscillating unit will enlarge all unstable regions of the combined system. However, the effect of the mass of the oscillating unit is insignificant to all unstable regions of the combined system. Existence of either the lower damper or the upper damper in the oscillating unit will make the combined system unstable once the spin rate exceeds the first critical speed of the disk. The effect of the viscous damping of the disk is favorable to all instability due to the disk modes only but will worsen the flutter-type instability between the oscillating unit and the reflected modes of the disk.

References

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