UNDULATIONS AND RIPPLES OF A THIN GRANULAR LAYER DUE TO VERTICAL VIBRATION

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Summary We performed an experiment of vertically oscillated granular layer and observed the global patterns, “undulations”. Several eigen modes are observed, which is reminiscent of the buckling and bending waves in an elastic rod. In our experiment “undulations” seem to be explained by continuum theory on the bending wave of an elastic plate, irrespective of material of the layer.

EXPERIMENT

Pattern formations of granular layer due to vertical vibration have been extensively studied since pioneering work by Faraday. Later there were many studies of pattern formation in vibrated granular layer [1] ∼ [13]. Here we consider global patterns of a granular layer, “undulations” as well as local patterns, “ripples”.

We performed an experiment of the ripples and undulations of a granular layer of thickness h in a vessel, which is a thin vertical acrylic container of width W(=8mm), height H(=91mm), and horizontal dimension L(=146, 91, 60, 46, and 30mm). We used several type of granular particles, sesame, glass beads, lead and steal particles, whose diameters d are 0.2mm ∼1.1mm. The observation was made by means of a high-speed camera. The vertical oscillation of the container was given by $z = a\sin(2\pi f t)$, where the frequency f and amplitude a were given by a function synthesizer and an amplifier under atmospheric pressure.

RESULT

Our observations are as follows: (i) When the acceleration amplitude $\Gamma = (2\pi f)^2 a/g$ (g: acceleration of gravity) is less than $\Gamma_0(\approx 2)$, no motion of granular particles relative to the container was observed. (ii) For $\Gamma = \Gamma_0 \sim \Gamma_1(\approx 4)$, the particles move up and down almost rigidly, where some convective motion is superposed, or ripples appear in a thin layer. (iii) At $\Gamma \approx \Gamma_1$, the upper free surface of the layer reveals undulations, but convective motion is still recognized. (iv) For Γ larger than Γ_1, a standing-wave-like motion of the granular layer is generated, with larger wave number for larger Γ or smaller $\eta \equiv h/d$. (v) Further increase of Γ yields a sequence of wave modes, as is shown in Fig. 1. We classify these typical patterns as A_n and S_n for anti-symmetric and symmetric deformations, respectively. Letters “S” and “A” denote patterns which are symmetric and anti-symmetric, respectively, with respect to the mid-point of the layer, and the subscripts reflect the number of undulations of the layer. The type S_0 denotes approximately rigid-body up-and-down motion. (vi) For Γ larger than $\Gamma_b(\approx 10)$, part of the layer shows burst-like behavior, whose position is not regularly spaced. From images of particle displacement$(g(t_1 + \delta t) - g(t_1))$, we observed the density waves.

DISCUSSION

Undulations are reminiscent of the buckling and bending waves (or flexural waves) of a thin elastic plate of Young’s modulus E [14]. In the case of noncohesive discrete systems, like those dealt with in this paper, the features of E needs a caution. However, the vibrating granular layer of moderate thickness suggests the presence of elastic property, which is manifested as density waves. All the data are plotted in Fig. 2, where the ordinate is the wave number and abscissa is a parameter which is derived from bending and buckling theory. From this we can see the sequence of data points seems to be fitted by a single curve $L/\lambda \propto \sqrt{a/h e_n}$, where λ is the wave length and e_n is the eigen value of the system that corresponds to each modes. Estimated Young’s modulus of the granular layer $E \approx 2.7 \times 10^4$Pa, which is quite small compared with that of the ordinary solids (i. e. $E \approx 2 \times 10^{11}$Pa for iron and $E \approx 3 \times 10^6$Pa for rubber.)

The above results can be qualitatively explained as follows. When the vertical vibration is applied to the granular layer the disturbance propagates both in the vertical and in the horizontal directions. If the dissipation in the vertical direction is small and the external Γ is small, the global patterns or “ripples” appear. On the contrary, If the energy dissipation in the vertical direction is large and external Γ is large, dilation would occur in the layer, and global patterns or “undulations” appear.

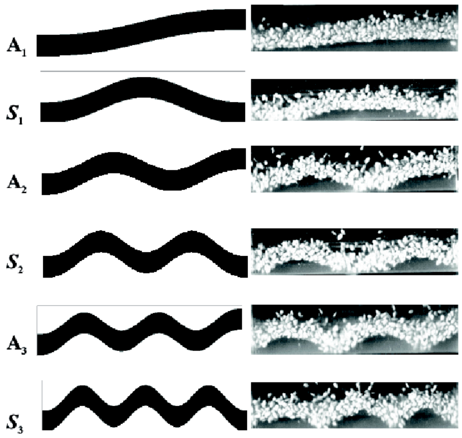

Fig. 1 Undulation of a thin granular layer. Right colimn is a corresponding example observed for an h=6.8mm layer of sesame :
$(A_1) f = 20$ Hz, $a = 2.6$ mm, $\Gamma = 4.2$;
$(S_1) f = 27$ Hz , $a = 2.5$ mm, $\Gamma = 7.4$;
$(A_2) f = 30$ Hz , $a = 1.4$ mm, $\Gamma = 5.1$;
$(S_2) f = 30$ Hz , $a = 2.0$ mm, $\Gamma = 7.3$;
$(A_3) f = 30$ Hz , $a = 2.3$ mm, $\Gamma = 8.4$;
$(S_3) f = 30$ Hz , $a = 2.6$ mm, $\Gamma = 9.6$.
Mode S_0 is an almost rigid-body up-and-down motion.

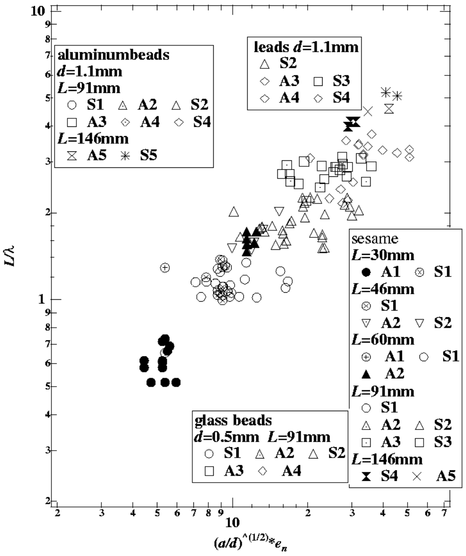

Fig. 2 Mode map of undulations.

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