WHY DO DOLPHINS HAVE CUTANEOUS RIDGES?

Peter W. Carpenter, Reza Ali
School of Engineering, University of Warwick, Coventry, CV4 7AL, U.K.

Summary

We present a simplified DNS study of 3D disturbances created in a laminar boundary layer with zero external pressure gradient by a line body force, located at the boundary-layer edge, and that varies harmonically with time and sinusoidally in the spanwise direction. We study walls, that are both rigid and compliant, whose undisturbed surfaces take the form of a small-amplitude static wave with crests normal to the freestream direction. This is analogous to the dolphin’s cutaneous ridges.

INTRODUCTION

The existence of small-scale static undulations on the outer surface of dolphin skin has been known for some time\(^1,2\). These features are known as cutaneous ridges or microscales. They appear as rings around the body, with wave crests aligned approximately normal to the freestream direction, and have wave-lengths of the order of 250 µm, about 5 to 8 times the wave (or ‘roughness’) height. No function, hydrodynamic or otherwise, has hitherto been suggested for the cutaneous ridges. Here we report on a simplified DNS study that was originally developed to study the effects of manufacturing imperfections in compliant walls used for drag reduction in marine applications. It will be shown that for rigid walls the static wall waves have a similar effect to conventional roughness, namely the boundary-layer disturbances (Tolnien-Schlichting waves) are destabilized and transition advanced. In contrast, for compliant walls, there is a range of ‘roughness’ wave-lengths for which the 3D TS waves grow less rapidly than over smooth walls.

COMPUTATIONAL METHODS

The velocity-vorticity method of Davies and Carpenter\(^3\) is used for our study. All variables are assumed to be non-dimensional with the freestream speed, \(U_\infty\), and boundary-layer displacement thickness, \(\delta^*\), used as reference. Only three primary variables are used, namely \(\omega_x, \omega_y, w\) that are respectively two components of the perturbation vorticity and the wall-normal perturbation velocity. These satisfy three governing equations of the form:

\[
\frac{\partial \omega_x}{\partial t} + \frac{\partial N_z}{\partial y} - \frac{\partial N_y}{\partial z} = \frac{1}{R} \nabla^2 \omega_x - \frac{\partial F_y}{\partial z}, \quad \frac{\partial \omega_y}{\partial t} + \frac{\partial N_x}{\partial z} - \frac{\partial N_z}{\partial x} = \frac{1}{R} \nabla^2 \omega_y + \frac{\partial F_y}{\partial x}, \quad (1a, b)
\]

\[
\nabla^2 w = \frac{\partial \omega_x}{\partial y} - \frac{\partial \omega_y}{\partial x}, \quad (1c)
\]

where \(x, y, z\) are respectively the streamwise, spanwise and wall-normal co-ordinates, \(R=U_\infty \delta^*/\nu\), \(F_y\) is the only non-zero component of the body force, and

\[
\mathbf{N} = \mathbf{\Omega} \times \mathbf{\dot{U}} + \mathbf{\omega} \times \mathbf{\dot{U}} + \mathbf{\omega} \times \mathbf{\dot{\omega}}
\]

(2)

where \(\mathbf{\dot{U}}, \mathbf{\dot{\Omega}}\) are the base-state velocity and vorticity, and \(\mathbf{\dot{\omega}}\) the perturbation velocity and vorticity. In [3] it is shown that subject to rather general conditions as \(z \to \infty\) eqs. (1a,b,c) are fully equivalent to the Navier-Stokes equations in primitive variables; it is also shown how the remaining perturbation components and pressure can be determined entirely in terms of the three primary variables. The method can be used to carry out full direct numerical simulations, but here we omit the last term on the right-hand side of eq. (2), thereby effectively linearizing the equations of motion.

One of the advantages of the method is that we can use a base state that is not, strictly, an exact solution of the Navier-Stokes equations. To obtain the base state in the present case we start with a constant-thickness boundary layer with a Blasius velocity profile. A steady-state perturbation to this base state corresponding to a wall in the form of a static wave:

\[
z = \eta_w(x) = k \sin(2\pi x / \lambda_w),
\]

(3)
If $k << 1$ the wall boundary conditions can be linearized, applied at $z = 0$ and written as

$$w = w_w = \eta, \quad u = u_w = -U'(0)\eta, \quad v = v_w = 0,$$

(4a,b,c)

where $\eta$ is the total vertical wall displacement (i.e., $\eta_w$ plus any elastic displacement). Eq (4a) can be applied directly. The no-slip conditions, Eq. (4b,c) are applied through rigorously derived integral conditions:

$$\int_0^\infty \! \omega_w \, dz = -u_w = \int_0^\infty \! \frac{\partial w}{\partial x} \, dz, \quad \int_0^\infty \! \omega_w \, dz = v_w + \int_0^\infty \! \frac{\partial v}{\partial y} \, dz.$$  

(5a,b)

Computationally this new base state is obtained by starting with the Blasius profile and gradually ‘ramping up’ the static wall wave until a steady well-behaved solution is obtained. We have also obtained an asymptotic, high-Reynolds-number solution which takes the form of an inner layer at the wall of thickness $O(\varepsilon^{1/3})$ where $\varepsilon = 1/R$. The streamwise velocity perturbation in this inner layer takes the form of an integral of an Airy function.

For the case of a compliant wall it is necessary to solve the equations of motion for the compliant wall interactively with eqs (1a,b,c) subject to conditions of continuity of velocity and tractive force at the interface. Here we use the plate-spring model for the compliant wall introduced by Carpenter & Garrad$^4$. This has the advantage of only introducing a single equation for the vertical wall displacement, $\eta$. Once the base state corresponding to the static wave has been established. It is driven by a body force of the form:

$$F_y = \int \! G \delta(x+y) \exp\{-a(x-x_f)^2 - b(z-z_f)^2\} \, dz.$$  

(7)

This corresponds to an approximation of a line vorticity source $\partial F_y/\partial z$ that is oscillating at frequency $\beta$ and varies sinusoidally in the spanwise direction with wave-number, $\gamma$. The exponential term is a numerical approximation to a delta function located at $(x_f, z_f)$. This form of vorticity source can be regarded as a rough approximation to components of freestream turbulence.

**RESULTS AND CONCLUSIONS**

For rigid walls the disturbances take the form of wave-trains of quasi-plane TS waves. The spanwise extent of individual wave-trains corresponds to the wave-number, $\gamma$, of the body force. In this case the presence of static waves always leads to a rise in growth rate and an increase in the unstable zone in $(R, \beta)$ space. Qualitatively, the results are in accord with previous studies of the effects of roughness on transition and boundary-layer instability. In particular there is fairly close agreement with the experimental data of Reshotko$^5$. For compliant walls it is known$^6$ that oblique waves grow more strongly than plane ones. Accordingly, the wave-trains take the form of oblique TS waves with the sign of the spanwise wave-number alternating in the spanwise direction. In this case for each combination of wall parameters $(k, \lambda_w)$ there is an optimum value of the driver’s spanwise wave-number, $\beta$, that corresponds to the most unstable case. It is found that for fixed static-wave amplitude, $k$, and Reynolds number, there is a range of values of $\lambda_w$ for which the growth rate of the TS waves is substantially reduced as compared with the smooth wall. This would result in a delay in the onset of transition for the wave wall. This phenomenon exhibiting a favourable effect of ‘roughness’ for compliant walls appears to be a new discovery. The favourable effect is greatest for a wave length, $\lambda_w \approx 5k$. This value is close to that found for the dolphin’s cutaneous ridges. So they may very well help the dolphin to maintain laminar flow. This provides for the first time an explanation of the possible favourable hydrodynamic effect of the dolphin’s cutaneous ridges.

**References**


