TRANSMISSION OF ELASTIC WAVES AND LOCALISED MODES IN COMPOSITE STRUCTURES

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Summary This work addresses the spectral analysis of elasticity problems in doubly periodic composite structures represented either by arrays of inclusions embedded into an elastic matrix or discrete lattice structures. Particular attention is paid to filtering properties, i.e. the presence of stop bands within the spectrum, and localised modes for composite structures containing defects.

First, we consider a problem of propagation of elastic waves through an elastic medium containing an infinite doubly periodic array of circular cylindrical inclusions. Each inclusion consists of a core of radius r_1 surrounded by a coating of width r_2. The materials of the core, the coating and the surrounding matrix are all isotropic and have different elastic properties and densities. Displacements and tractions remain continuous across each interface. We assume here that the cylinders are infinitely long and all aligned perpendicular to the plane of propagation. In this case the problem is decoupled into two, in-plane and out-of-plane, problems. We analyse a Bloch-Floquet problem posed in an elementary cell of an infinite doubly periodic array. In particular, we show that the parameters of the coating can be tuned in such a way that, in the long-wave approximation, the elastic system responds as a homogeneous medium without inclusions; in this case the inclusions are classified as 'neutral', and their presence does not affect the value of the effective refractive index. However, for higher frequencies such structures exhibit stop bands and possible localisation of eigen-modes within either the coating or the inclusion core. The mathematical model used for analysis of this class of problems is based on the concept of multipole approximations as well as the transfer matrix approach discussed in detail in [1], [2].
Further, we note that it is efficient to model localised eigen-modes using approximations involving discrete lattice structures. As outlined in [3], the presence of 'defects' within a macro-cell of an infinite periodic lattice may generate a highly localised eigen-mode and low frequency stop bands for Bloch-Floquet elastic waves. This concept is developed in the present work, together with explicit asymptotic estimates for the eigen-frequencies of localised modes that exist in the densely-packed composite structures containing thin ligaments.
Since the modelling of discrete dynamic lattice structures is more straightforward compared to the analysis of continuum composites, it is important to be able to approximate the high-contrast densely packed composite structures by discrete lattices. This issue is also addressed in the present work.

References

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