AXISYMMETRIC FORCE SOLUTION FOR A SEMI-INFINITE CUBIC SOLID

Charles Ruimy*, Marc Dahan**

*UHA, ERT Mécanique des Matériaux sous Grande Vitesse, F 68000 Mulhouse, France

**UFC, Laboratoire de Mécanique Appliquée, F 25000 Besançon, France

Summary An exact three-dimensionnal analysis is developped for an axisymmetric loading on the surface of a half-space composed by an anisotropic cubic medium. The general solution is given for a surface concentrated loading by exact integral expressions. Isovalue curves of stress permit easily to compare the anisotropic results with the isotropic ones.

BASIC EQUATIONS

We consider a homogeneous elastic half-space *z>0*, referred to a fixed rectangular Cartesian coordinate system (*Oxyz*). We assume that the medium is characterized by cubic anisotropy, with the *z*-axis as elastic symmetriy axis. Let arbitrary axisymmetric normal stress be prescribed all over the boundary *z=0*. We need to find the complete field of stress and displacements in the half-space.

By using cylindrical coordinates (r,θ,z), we designate the components of the displacement field (u_r,u_θ,u_z), the components of the strain tensor $(\varepsilon_r,\varepsilon_{\theta\theta},\varepsilon_{zz},\varepsilon_{\theta z},\varepsilon_{rz},\varepsilon_{r\theta})$ and the non-zero components of the stress tensor $(\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz},\sigma_{rz})$.

We use some results obtained in [3], with the same notations, and we show that the introduction of a potential function permits to reduce to one the number of independant equations (compatibility and equilibrium). It is a differential equation of the 4th order which has the solution

$$\varphi(r,z) = \int_0^\infty (Ae^{s_1 mz} + Be^{s_2 mz} + Ce^{-s_1 mz} + De^{-s_2 mz})J_0(mr)mdm$$

The coefficients *A, B, C, D* are determined by the boundary conditions and the regularity conditions at infinity. Then, the formulae for stresses take the form

$$\sigma_{zz}(r,\theta,z) = \int_0^\infty (Cs_1g_1e^{-s_1 mz} + Ds_2g_2e^{-s_2 mz})m^4J_0(mr)dm$$

$$\sigma_{rz}(r,\theta,z) = \int_0^\infty (Cp_1e^{-s_1 mz} + Dp_2e^{-s_2 mz})m^4J_1(mr)dm$$

RESOLUTION

We consider the case of a uniform normal pressure distribution p_0 on a circle (radius r_0) applied on the medium surface $z = 0$. The Hankel transformation of loading function is

$$p^H(m) = p_0r_0J_1(mr_0)$$

and we obtain successively $A = B = 0$,

$$C = \frac{k(m)p_2}{(p_2s_1g_1 - p_1s_2g_2)}$$

$$D = -\frac{k(m)p_1}{(p_2s_1g_1 - p_1s_2g_2)}$$

with

$$k(m) = -p_0r_0J_1(mr_0)/m^4$$

Thus, by integration of the expressions of σ_{zz} and σ_{rz}, it is possible to obtain the field of stress in the complete medium. The following curves present the results for two cubic materials (copper and molybdenum). The cupper has a behavior very different of the isotropic one (in dotted line). The molybdenum is similar. This is consistent with the values of the elastic coefficients of each material.

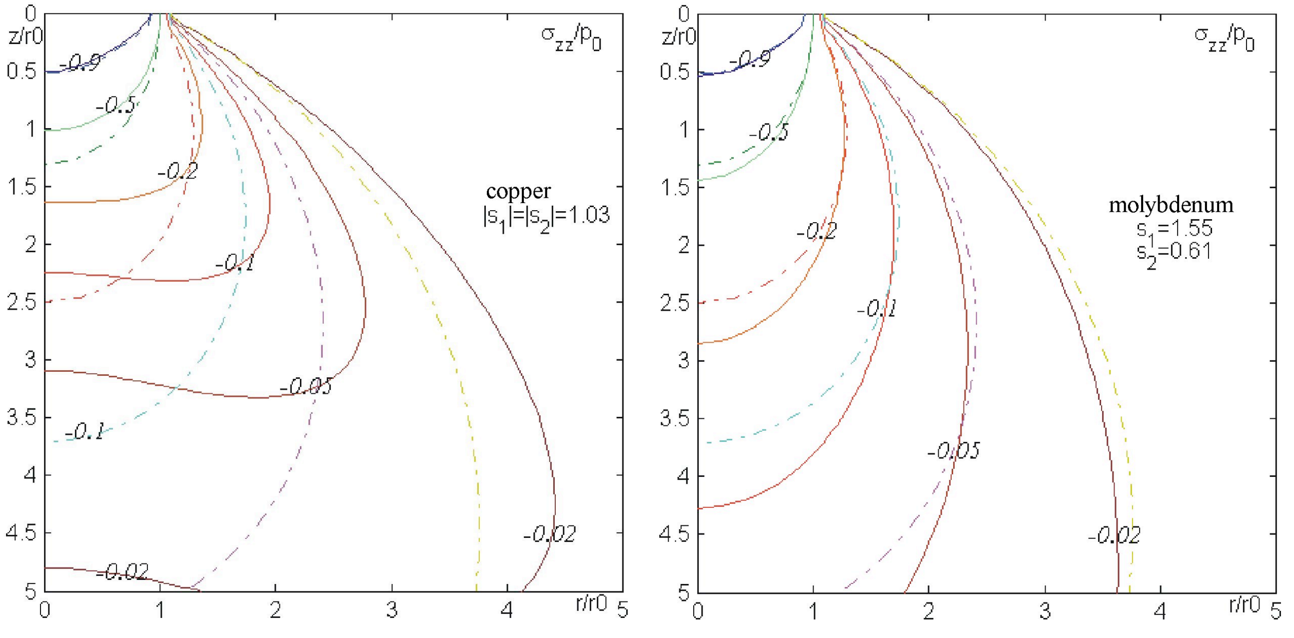
When the surface loading is reduced to a concentrated force *P*,

$$k(m) = -\frac{P}{2\pi m^3}$$

and thus we obtain explicit solutions for the stress and displacement in the half-space.

CONCLUSIONS

From fundamental equations of solid mechanics (equilibrium equations, compatibility conditions, generalized Hooke's law), we introduce a new potential function from the stresses and we are able to express the displacements only with this function. This method is usable for all axisymmetric loadings in the surface or in the interior of an infinite or semi-infinite cubic medium. It gives new solutions for contact elastic problems between anisotropic materials. Today, we have at our disposal only explicit solutions for isotropic materials [4] and transversely isotropic materials [1,2].



[1] Dahan M., Predeleanu M.: Solutions Fondamentales pour un Milieu Elastique à Isotropie Transverse. *ZAMP* **31**: 413-424, 1980.

[2] Fabrikant V.I.□New Approach to Mixed-mixed Problems in Linear Elasticity. *Eur. J. Mech., A/Solids* **16**□595-618, 1997.

[3] Ruimy C.□Chargements Axisymétriques sur un Milieu à Isotropie Tétragonale. *PhD Thesis*, University of Franche-Comté, Besançon, 2002.

[4] Ufliand Ia.□The Contact Problem of the Theory of Elasticity for a Die, Circular in its Plane, in the Presence of Adhesion. *Prikladnaia Matematika i Mekhanika* **20**□578-587,1956.