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THE INFLUENCE OF REMOTE STRESSES ON THE NEAR CRACK TIP STRESS FIELD

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Summary Two methods of finding the near crack tip field can be specified. While one consists of retaining only terms valid in the infinitesimal vicinity of the crack tip, the other is obtained by infinitely extending the whole crack. For one crack these approaches are equivalent. For interacting cracks or cracks with loads acting on their planes, we get a loss of uniqueness. This occurs for elastic and elastic-plastic materials. In the paper we present examples and a discussion which enables to distinguish the differences. From this point of view also the kinked crack tip is analyzed in the case of a small kink or small kink angle. Not to complicate the discussion only mode III cracks are studied.

Preliminaries

Let us assume a Cartesian coordinate system (x_1, x_2, x_3) with the origin at the crack tip and the negative part of x_1 axes aligned with the crack. In the case of mode III loading, the displacement vector $\mathbf{u}$ is of form

$$\mathbf{u} = [0, 0, w(x_1, x_2)], \tag{1}$$

where $w(x_1, x_2)$ is the displacement parallel to the x_3 axis. The displacement can be expressed as an imaginary part of a complex analytic function $f(z)$, i.e $w(x_1, x_2) = (1/\mu)Im[f(z)]$, where $z = x_1 + \imath x_2$. Thus, for the stresses in an elastic body we have

$$\sigma_{23} + \imath\sigma_{13} = f'(z), \tag{2}$$

In the perfectly plastic region at the crack tip [4] we obtain

$$\sigma_{23}(r,\theta) + \imath\sigma_{13}(r,\theta) = k\, e^{-\imath\theta}. \tag{3}$$

In an elastic body the asymptotic stresses at the crack tip are

$$\sigma_{23} + \imath\sigma_{13} \to \frac{K_{III}}{\sqrt{2\pi z}}. \tag{4}$$

The problem of an infinite crack in an infinite body with inverse square root singularity has the form eq.(4) (i.e. $f'(z) = K_{III}/\sqrt{2\pi z}$)

Load acting on crack plane

In the case of loads

$$\sigma_{23}(x_1, 0) = \frac{ka}{x_1 - a}, \tag{5}$$

acting on the crack plane, we obtain two stress distributions [1]

$$\sigma^0_{23} + \imath\sigma^0_{13} = k\frac{a}{\sqrt{z}(\sqrt{z}+\sqrt{a})} \tag{6}$$

and

$$\sigma_{23} + \imath\sigma_{13} = \frac{k\sqrt{a}}{\sqrt{z}}\left(\frac{\sqrt{a}}{\sqrt{z}+\sqrt{a}} + 1\right), \tag{7}$$

which differ only by the asymptotic decay ratio in infinity. It is the same on the crack plane because both solutions satisfy the same BVP on the crack, also on the 'infinite' end of the crack.

Two parallel cracks

Let two parallel infinite cracks be at distance $2h$ from each other and the stresses on the outside of the cracks vanish in infinity such as $z^{1/2}$. The inverse stress function is given by [2]

$$f'(z) = \sigma_{23} + \imath\sigma_{13} = \frac{K_{III}}{\sqrt{h}}\ \frac{\sqrt{z_1 - 1}}{z_1} \tag{8}$$

for

$$z = \frac{h}{\pi}(z_1 + \ln(1 - z_1)) \tag{9}$$

The above result can be added to any loaded case to obtain a different stress distribution satisfying the same BVP.

Kinked crack

The exact solution for a mode III crack was given by [3], where the conformal mapping was obtained using the Schwarz-Christoffel formula. Quite often the main most cumbersome problem in the use of this formula is the determination of the unknown constants. There they were chosen for the sake of simplifying the integration. In conformal mappings there are three arbitrary constants. Luckily, in this case of an infinite crack, they can be analytically obtained to satisfy the condition that the two cases: of a vanishing kink and a vanishing kink angle result in the same BVP of one straight crack. For a kink angle β :

$$f'(z_1) = \frac{\imath}{1+\phi h\beta}(z_1 + h\beta)^{\beta}(z_1 - h\beta)^{-\beta} \tag{10}$$

where

$$z = -(z_1 - h\phi)^{1+\beta}(z_1 + h\phi)^{1-\beta}, \qquad \phi = (1+\beta)^{-(1+\beta)/2}(1-\beta)^{-(1-\beta)/2} \tag{11}$$

Small scale yielding

In the case of small scale yielding for one crack we can obtain different stress distributions (they both have vanishing remote stress but of different order)[1]:

$$F_1(\omega) = A_1(\frac{1}{\omega^2}+1) + \imath A_2(\frac{1}{\omega^3}-\omega), \qquad F_1(\omega) = A_1(\frac{1}{\omega^2}+1) + A_3(\frac{1}{\omega^4}+\omega^2). \tag{12}$$

where

$$F_1(\omega) = f'^{-1}(\omega), \qquad \omega = \sigma_{23} - \imath\sigma_{13} \tag{13}$$

For two parallel cracks different solutions can be obtained even with the same order in infinity [2]:

$$\begin{aligned} \omega(\varphi)\, F(\omega(\varphi)) \;&=\; -\frac{h}{\pi}\sqrt{P(\varphi)}\left(\frac{\sqrt{1/4-b^2}}{\sqrt{1/4-\varphi^2}}\ln\frac{4(\varphi\sqrt{1/4-b^2}-b\sqrt{1/4-\varphi^2})^2}{\varphi^2-b^2} - \ln\frac{\varphi-b}{\varphi+b}\right) \\ &+\; C\sqrt{P(\varphi)}, \qquad \varphi(\omega) = \frac{\sqrt{(1-S'^2\omega^2)(S'^2-\omega^2)}}{(1-S'^2\omega^2)+(S'^2-\omega^2)}. \end{aligned} \tag{14}$$

CONCLUSIONS

To ensure uniqueness of either asymptotic crack tip stresses or an infinite crack problems it is suffices to demand finite displacements at the crack tip and attenuation of stresses in infinity. In other cases it is not enough and can lead to multiple solutions of the same problem. Multiple in the sense that the same BVP can have two solutions, which in the limit, at the crack tip, give different Stress Intensity Factors (see eq.(6, 7)). Or, in the case of a kinked crack, a vanishing kink is not consistent with vanishing kink angle. Thus, there is a need to impose some additional condition (eg. rate and order) relevant to the considered physical problem.

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