

## SLENDER BODY THEORY APPROACH TO NONLINEAR SHIP MOTIONS

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**Summary** A solution method for the time-domain investigation of nonlinear heave and pitch motions of ships in head waves is presented. By virtue of the slender body assumption, the three-dimensional flow problem is decomposed into a series of two-dimensional problems. The major nonlinearities are induced by motions with large amplitude and by the physical nature of the free water surface.

### INTRODUCTION

The accurate prediction of large amplitude ship motions in severe seas poses still a delicate problem in the field of fluid-structure interaction. While three-dimensional panel methods have reached the state of maturity in linear seakeeping analysis, the original problem, governed by strongly nonlinear boundary conditions, is far from being solved efficiently. The principal nonlinearities are associated with the variable wetted surface of the ship hull and with the nonlinear hydrodynamic conditions on the free surface.

Over a period of years, the problem of solving an instantaneous nonlinear boundary value problem has been circumvented by integrating the pressure of the undisturbed incoming waves over the actual wetted surface (to obtain the so-called nonlinear Froude-Krylov force) while treating a linear radiation/diffraction problem. The negligence of nonlinear hydrodynamic effects was justified by the different magnitudes of the linear Froude-Krylov and the radiation/diffraction forces. However, it has been proved in an exemplary study [2], that the nonlinear hydrodynamic effects can attain the same order of magnitude as the contributions from the nonlinear Froude-Krylov force. In consequence, a consistent investigation of nonlinear ship motions must consider both, geometric and hydrodynamic nonlinearities.

A promising approach, with regard to efficiency, is provided by the so-called  $2D+t$  theory [1] which has been successfully applied to the prediction of high-speed craft wave resistance and ship bow waves. Following the lines of slender body theory, the three-dimensional flow problem may be decomposed into a series of consecutive two-dimensional problems. Until present, only the forced motion problem has been investigated with  $2D+t$  theory. This paper generalizes the approach to free body motions.

### STATEMENT OF PROBLEM

#### Slenderness and Scaling

The rigid body motion of the ship and the wave flow will be described with respect to an inertial reference frame. The  $(x, y)$  plane defines the free water surface at rest and the  $z$ -axis is pointing vertically upward out of the fluid domain. We consider a ship advancing in head waves with forward velocity  $U$  in direction of the  $x$ -axis. The ship is free to heave and pitch. All other modes of motion are constrained.

The characteristic hull dimensions are length  $l$ , draft  $d$ , and maximum beam  $b$  of the ship. We assume that the ship is slender, i.e.,  $b \simeq d \ll l$ . Then, the components of the outward surface normal vectors are  $n_X = \mathcal{O}(b/l) = \mathcal{O}(\epsilon)$ ,  $n_Y = \mathcal{O}(1)$  and  $n_Z = \mathcal{O}(b/d) = \mathcal{O}(1)$ . For many ships, like container ships, frigates or cruise vessels,  $d/l = \epsilon = \mathcal{O}(10^{-1})$ . The parameter  $\epsilon$  is referred to as the slenderness parameter. The boundary value problem describing the ship flow will be linearized with respect to the slenderness parameter in order to reduce the physical dimension of the problem. However, the nonlinear character of the boundary conditions will be retained.

#### Boundary Value Problem

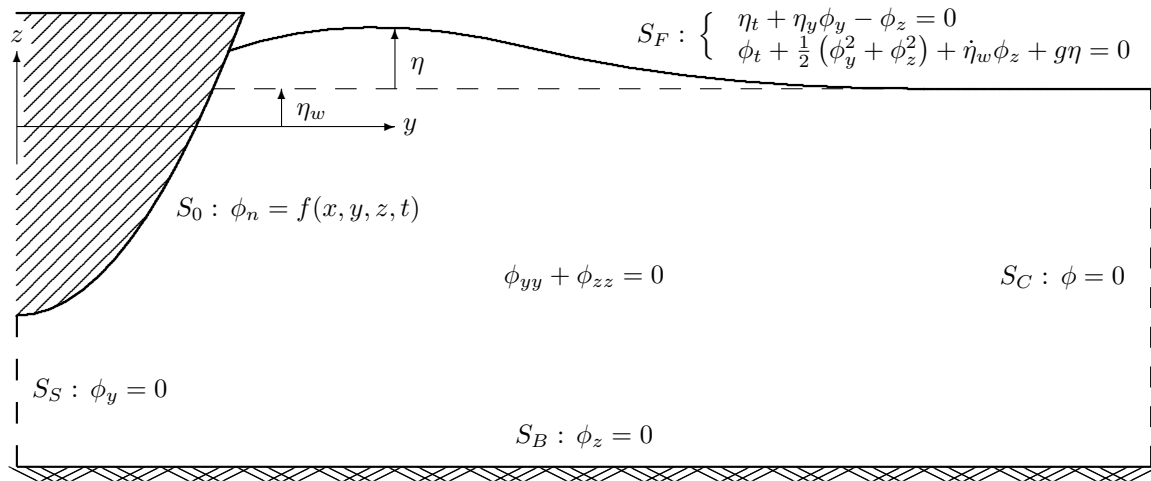
We imply, that the wave patterns around the ship are described sufficiently by an inviscid, incompressible and irrotational flow. Then, the flow may be determined by a potential  $\phi$  which has to satisfy Laplace equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0. \quad (1)$$

Order of magnitude analysis reveals that the first term in (1) is of  $\mathcal{O}(\epsilon^2)$  in comparison to the latter. It is therefore neglected in the sense of slender body theory. The total flow potential is given by the potential of the unperturbed wave  $\phi_w$  and the ship induced disturbance potential  $\phi$ . The unknown disturbance potential must satisfy the zero-flux condition on the body surface  $S_0$

$$\frac{\partial \phi}{\partial n} = \nabla \phi \cdot \vec{n} = -U n_X - \left[ U \theta + \dot{z}_G - \dot{\theta}(x - x_G) - \dot{\eta}_w \right] n_Z, \quad (2)$$

where  $\vec{n}$  is the normal vector in the  $(y, z)$  cross plane, pointing out of the fluid domain. The heave velocity is denoted by  $\dot{z}_G$ , the pitch angle and velocity by  $\theta$  and  $\dot{\theta}$ , respectively. The vertical velocity of the incoming wave is given by  $\dot{\eta}_w$ .



**Figure 1.** Boundary conditions for axis symmetric problems.

The kinematic and dynamic conditions on the free surface  $z = \eta + \eta_w$  yield

$$\eta_t + \eta_y \phi_y - \phi_z = 0, \quad (3)$$

$$\phi_t + \frac{1}{2} (\phi_y^2 + \phi_z^2) + \dot{\eta}_w \phi_z + g\eta = 0. \quad (4)$$

In order to obtain a finite and bounded computational domain, additional boundary conditions are enforced at the symmetry plane  $S_S$ , at the bottom plane  $S_B$ , and at a vertical control plane  $S_C$  far away from the body, Figure 1.

The instantaneous boundary value problem is further decomposed into an initial value problem for the temporal evolution of the free surface quantities  $(\phi, \eta)$  and a boundary value problem for  $(\phi$  on  $S_0, \phi_n$  on  $S_F)$  fixed in time, which is solved by a standard Boundary Element Method. Equation (2) is used to provide the flux on the body boundary  $S_0$ . The free surface quantities are updated from (3) and (4) by the Mixed Eulerian-Lagrangian Method [3].

### Integration of Body Motions

In order to resolve the three-dimensional flow, two-dimensional boundary value problems have to be solved at 20 to 30 sections along the ship hull. The hydrodynamic forces and moments are computed from the pressure distribution over the ship hull, where the pressure is obtained from the total flow potential by Bernoulli's equation.

Instability may arise in the numerical integration scheme due to the particular nature of the hydrodynamic force. From linear seakeeping theory it is well known that the total hydrodynamic force contains terms proportional to the position, velocity and acceleration of the ship. General numerical stability of ordinary differential equations, however, requires that all terms proportional to the highest derivative, i.e. the acceleration, have to be isolated.

## RESULTS AND CONCLUSIONS

Stable numerical solutions of heave and pitch motions are obtained at a reasonable time step size when the ship induced flow potential is further decomposed into the contributions depending on acceleration, velocity, and displacement like in linear theory. Due to nonlinearity, the decomposition is no longer unique and the coefficients for added mass, radiation damping and hydrostatic restoring have to be updated in every time step. The results of the nonlinear analysis show asymptotic behavior to the linear solution when the height of the incoming waves is decreased.

The application of  $2D + t$  theory to ship motions allows to treat the nonlinear problem eluding the complexity of three-dimensional computations. The present analysis of heave and pitch motions may be extended consistently to ship roll motions at moderate cost. This will enable the investigation of parametric rolling in head and quartering seas which represents a significant risk to modern day ships.

### References

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