Many-sphere hydrodynamic interactions:
weak convective inertia effects

I. T. Pieńkowska

Institute of Fundamental Technological Research
Świętokrzyska 21, 00-049 Warsaw, Poland

The motivation for this work is to analyse the effects of weak convective inertia of the fluid on the many-body hydrodynamic interactions. We consider a finite number N of rigid spheres, held rigidly in the incompressible, viscous fluid, having a uniform velocity U at infinity. The paper deals with weak inertia effects, influencing the hydrodynamic forces F_j, $j = 1, \ldots, N$, exerted by the fluid on the spheres, in the régime of small Reynolds numbers Re, based on the radius a of the spheres, the kinematic viscosity v of the fluid and the magnitude U of the fluid velocity. In particular, we will examine the violation, due to the inertia effects, of the Stokes symmetry, characterizing the Stokes many-body hydrodynamics interactions and the respective hydrodynamic forces, experienced by the spheres.

The range of the considered hydrodynamic interactions is specified by means of the following parameters:

(i) $\sigma = a/R$, where R is the typical distance between the centres of the spheres, $\sigma < \tfrac{1}{2}$,

(ii) $\mathrm{Re} = aU/v$, $\mathrm{Re} < 1$.

The problem is considered on the basis of the Oseen equations. The presence of the spheres in the flow is taken into account by means of the so-called induced forces f_j, distributed on the surfaces of the spheres. The velocity field of the fluid is expressed in terms of the integral equations, involving the inertia-dependent Green tensor. Using the no-slip boundary conditions on the surfaces of the spheres, we have obtained a set of N integral equations, relating the induced forces f_j to the velocities V_j of the spheres with respect to the unperturbed fluid. The many-sphere hydrodynamic interactions are represented in terms of the multiple scattering processes. The multiple scattering events are described by means of the so-called self- and mutual interaction tensors, concerning, respectively, the hydrodynamic interactions of an individual sphere with the surrounding fluid, and the mutual interactions between two spheres. In terms of these interaction tensors, such properties of the hydrodynamic interactions, as the long range, non-locality, and non-additivity, are discussed.

The hydrodynamic forces F_j are obtained in the form of an infinite series, describing the allowed sequences of the hydrodynamic interactions. That series enables us to calculate the regarded forces within the assumed approximation with respect to the small parameters σ , and Re. We have distinguished different régimes of the hydrodynamic interactions and the resulting forces F_j for small, and respectively, large values of the parameter RU/v (the interactions close to, and far from the Stokes conditions). As examples, we will consider the drag, and side forces, exerted on three spheres, placed in line, in the transversal and parallel directions with respect to U.

We will apply the presented solutions to the Oseen boundary value problem to examine the analogous Navier-Stokes boundary value problem, at small data, in the sense of Finn (Robert Finn, ARMA **19**, 363 (1965)).