ON SEPARATION OF EIGENFREQUENCIES IN TWO-MATERIAL STRUCTURES

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Summary We present a method to maximize the separation of two adjacent eigenfrequencies in structures with two material components. The method is based on finite element analysis and topology optimization where an iterative algorithm is used to find the optimal distribution of the materials. Results are presented for vibration problems governed by the 2D scalar wave equations.

INTRODUCTION

One strategy for passive vibration control of mechanical structures is to design the structures so that eigenfrequencies lie as far away as possible from the excitation frequencies. This paper exploits the possibility for using the method of topology optimization to maximize the separation of two adjacent eigenfrequencies in structures with two material components. This study is restricted to 2D structures where the vibrations are governed by the scalar wave equation.

THE 2D SCALAR CASE

Here we show results related to the 2D scalar case. Results also including the 1D scalar case can be found in [1].

Model

The 2D scalar time-reduced wave equation (Helmholtz equation) is given by:

$$\boldsymbol{\nabla}^T(A(x,y)\boldsymbol{\nabla}w) + \omega^2 B(x,y)w = 0, \tag{1}$$

where the problem dependent material coefficients A and B can vary in the 2D plane (x, y). By changing the two coefficients A and B we study different structural vibration problems. Letting $A = 1$ and $B = \rho/T$ enable us to analyze the membrane problem where $\rho(x)$ is the density and T is the uniform tension (force per area). Alternatively with $A = E/(2(1+\nu))$, where $E(x)$ is Young's modulus and $\nu(x)$ is Poisson's ratio, and with $B = \rho(x)$ being the density, (1) governs out-of-plane shear vibrations of a thick elastic body.
To solve the wave equation with in-homogeneous coefficients we apply a standard Galerkin finite element discretization of equation (1) and the boundary conditions, which leads to the discrete eigenvalue problem:

$$\mathbf{K}\boldsymbol{\phi} = \omega^2\mathbf{M}\boldsymbol{\phi}, \tag{2}$$

that has $(\omega_i, \boldsymbol{\phi}_i)$ as the i'th eigensolution (frequency and vector), and $\mathbf{M}$, $\mathbf{K}$ are system matrices given as:

$$\mathbf{K} = \sum_{e=1}^{N} A_e\mathbf{k}_e, \qquad \mathbf{M} = \sum_{e=1}^{N} B_e\mathbf{m}_e, \tag{3}$$

where the summations should be understood in the normal finite element sense, with $\mathbf{k}_e$ and $\mathbf{m}_e$ being element matrices.

OPTIMIZATION

When we optimize a 2D domain with respect to maximizing the gap between eigenfrequencies there are a number of extra difficulties we must deal with. The primary source of the difficulties is the possibility of multiple eigenfrequencies. The multiple eigenfrequencies can be calculated without difficulty using e.g. the subspace iteration method [2]. The objective for the optimization is given by

$$\text{maximize} \quad J = \omega_{n+1} - \omega_n, \tag{4}$$

where the gap between the eigenfrequency of order $n+1$ and n is maximized. If the eigenfrequencies of order $n+1$ and n are both distinct eigenpairs, with squared eigenfrequencies ω_{n+1}^2 and ω_n^2 and corresponding eigenvectors $\boldsymbol{\phi}_{n+1}$ and $\boldsymbol{\phi}_n$, no problems arise and we use the objective (4) directly since the sensitivities of the squared eigenfrequency with respect to a design parameter t_e are then given by:

$$\frac{d\omega^2}{dt_e} = \boldsymbol{\phi}^T\left(\frac{d\mathbf{K}}{dt_e} - \omega^2\frac{d\mathbf{M}}{dt_e}\right)\boldsymbol{\phi}, \tag{5}$$

assumed that the eigenvector have been normalized so that $\boldsymbol{\phi}^T\mathbf{M}\boldsymbol{\phi} = 1$. In the case of multiple eigenvalues we cannot use (5) to find the sensitivities. The extended method is presented in [3] and used more recently in [4], and with this

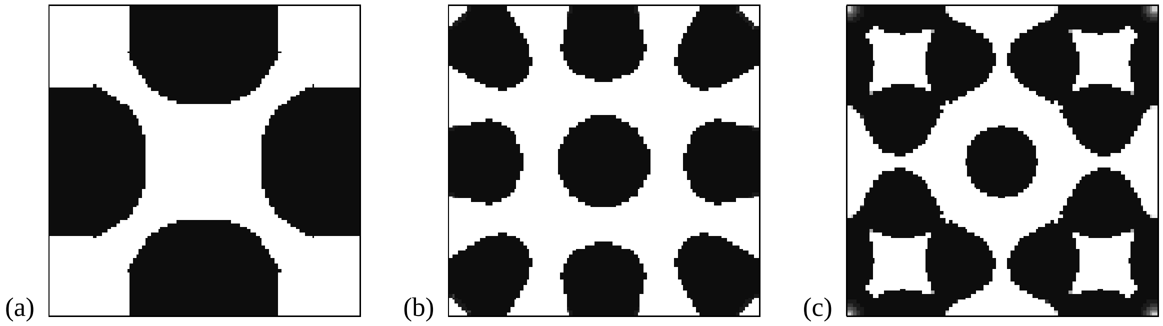

Figure 1. Eigenfrequency optimization of a 2D domain with two different materials, the design domain is a square that have no supports (free boundary conditions); (a) the result when maximizing the gap between 2nd and 3rd eigenfrequency, (b) 8th and 9th eigenfrequency, (c) 12th and 13th eigenfrequency.

method it is now possible to find the sensitivities of multiple eigenfrequencies. There is however still a problem because the sensitivities are given for specific eigenvectors that vary for each design parameter. It is therefore difficult to solve the optimization problem as formulated in (4). As an alternative formulation we propose to use a double bound formulation. The standard bound formulation is used to reformulate a min-max problem. Instead of minimizing the maximum value of a given quantity, a new variable is introduced which is minimized subject to the constraint that the value of the given quantity should be less than this variable. The present optimization problem of maximizing the gap between two eigenfrequencies is reformulated as:

$$\begin{array}{rll} \max_{t_e} & J = C_1 - C_2 & \\ \text{subject to:} & \omega_{n+i} \geq C_1 & i \in [1, n_u] \\ & \omega_{n+1-j} \leq C_2 & j \in [1, n_l] \end{array} \tag{6}$$

Penalization

The basis of topology optimization with a material interpolation scheme is to assign constant material properties to each element in the finite element model and then to associate these material properties with continuous design variables. The simplest interpolation of the material coefficients A and B when using two materials is done using a linear approach:

$$A_e(t_e) = A_1 + t_e(A_2 - A_1), \qquad B_e(t_e) = B_1 + t_e(B_2 - B_1), \tag{7}$$

where t_e is the element design parameter which takes values between 0 and 1. This is however not appropriate when optimizing eigenfrequencies. In [5] it was noted that the important aspect is not the interpolation of the stiffness (here coefficient A) or the interpolation of the mass (here coefficient B) but the interpolation of the eigenfrequency. The squared eigenfrequency ω^2 is by the Rayleigh quotient given as "stiffness divided by mass". A first choice would then be to make an interpolation of the squared eigenfrequency which is a linear function of the design parameter t_e. This will however still not give a 0-1 design i.e. a design with no intermediate material. We must therefore make a penalization that will drive the design to a final 0-1 result. It is possible to make an interpolation/penalization function that will work for an objective where we want to maximize an eigenfrequency but this will not work for a minimization of an eigenfrequency. In the objective of (6) we both have to maximize one eigenfrequency and at the same time minimize a second eigenfrequency this cannot be achieved by one penalization function. To overcome this problem we apply a method where the sensitivities related to the constraints with C_1 in (6) are calculated using one penalization function and the sensitivities related to the constraints with C_2 are calculated using a second penalization function.

EXAMPLES

In figure 1 we show the results of maximizing the gap between three different sets of eigenfrequencies of a square domain, black and white corresponds to the two different materials.

References

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