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COUPLING BETWEEN PERMEABILITY AND DAMAGE : A MICROMECHANICAL APPROACH

Luc Dormieux*, Djimedo Kondo**
*LMSGC, CNRS UMR 113, Ecole Nationale des Ponts et Chaussées, France
**LML, CNRS URA 1441, Université des Sciences et Technologies de Lille, France

Summary A self-consistent scheme is used in order to determine the permeability of a cracked porous medium. For weak values of the permeability of the uncracked porous matrix, the order of magnitude of the permeability increases beyond a critical threshold of the crack density parameter. The micromechanical model also shows that both the evolution of crack opening and the crack propagation are controlled by Terzaghi's effective stress which therefore captures the coupling between permeability and mechanical loading.

a self-consistent estimate of the macroscopic permeability

Experimental evidences show a very strong coupling between damage and permeability. This paper proposes to model this phenomenon within a micromechanical framework that also allows to quantify the influence of a mechanical loading on permeability.

The starting point is a classical idea in rock physics ([1]) which consists in approximating the real fluid flow within a crack by the Poiseuille flow which takes place between two parallel planes (half-distance c). This leads to introduce a fictitious porous medium with permeability $\boldsymbol{k}_c$, equivalent to the real crack as regards the relation between flow and pressure gradient. Denoting the unit normal to the crack by $\underline{n}$ and the second order identity tensor by $\mathbf{1}$, $\boldsymbol{k}_c$ reads :

$$\boldsymbol{k}_c = k_c\mathbf{1} + (k_p - k_c)\underline{n}\otimes\underline{n} \quad \text{with} \quad k_c = \frac{c^2}{3} \tag{1}$$

where k_p denotes the isotropic permeability of the uncracked porous medium. We now look for the effective intrinsic permeability tensor $\boldsymbol{K}$ of a representative elementary volume (r.e.v.) Ω made up of the porous material with permeability k_p and of a set of cracks. $\boldsymbol{K}$ relates the macroscopic filtration vector $\underline{Q}$ to the macroscopic pressure gradient $\underline{\nabla P}$:

$$\underline{Q} = -\frac{\boldsymbol{K}}{\mu}\cdot\underline{\nabla P} \tag{2}$$

where μ is the fluid viscosity. Replacing the cracks by the fictitious porous material with permeability $\boldsymbol{k}_c$ introduced in (1), the r.e.v. considered at the microscopic scale appears as a heterogeneous Darcy material : the local permeability is $\boldsymbol{k}(\underline{z}) = k_p\mathbf{1}$ in the uncracked phase and $\boldsymbol{k}(\underline{z}) = \boldsymbol{k}_c$ in the cracks. Note that $\boldsymbol{k}_c$ depend on the crack orientation and opening through $\underline{n}$ and c.

Let $\underline{q}(\underline{z})$ and $\underline{\mathrm{grad}}_z\, p\,(\underline{z})$ denote the filtration vector field and the pressure gradient field within the r.e.v., i.e. at the microscopic scale. They are related by the heterogeneous Darcy Law (3b). The macroscopic filtration vector is related to the microscopic field $\underline{q}(\underline{z})$ by the average rule $\underline{Q} =< \underline{q} >$. In turn, the microscopic pressure field $p(\underline{z})$ is subjected to the standard Hashin boundary conditions (3c) associated with the macroscopic pressure gradient $\underline{\nabla P}$

$$(\Omega):\ \mathrm{div}\,\underline{q} = 0 \quad (a) \quad ; \quad (\Omega):\ \underline{q} = -\boldsymbol{k}(\underline{z})\cdot\underline{\mathrm{grad}}_z\, p \quad (b) \quad ; \quad (\partial\Omega):\ p = \underline{\nabla P}\cdot\underline{z} \quad (c) \tag{3}$$

where (3a) expresses the fluid mass balance equation. The linearity of the solution $(p,\underline{q})$ to (3) with respect to $\underline{\nabla P}$ is classically expressed through the concept of localization tensor $\boldsymbol{A}(\underline{z})$ which relates the local and the macroscopic pressure gradients :

$$\underline{\mathrm{grad}}_z\, p\,(\underline{z}) = \boldsymbol{A}(\underline{z})\cdot\underline{\nabla P} \tag{4}$$

Combining (3b) and (4) with (2) yields a general micromechanical interpretation of the macroscopic permeability :

$$\boldsymbol{K} =< \boldsymbol{k}\cdot\boldsymbol{A} >= f_p k_p \mathbf{1}\cdot < \boldsymbol{A} >_p + \sum_j f_j \boldsymbol{k}_c^j \cdot < \boldsymbol{A} >_j \tag{5}$$

where f_p (resp. f_j) denotes the volume fraction of the uncracked phase (resp. of crack $\mathcal{C}^j$) and $<>$ (resp. $<>_\alpha$) is the average over Ω (resp. over the α-phase). The various homogenization schemes aim at estimating the phase averages of $\boldsymbol{A}$, depending on the morphological properties. The self-consistent scheme is selected here since it seems reasonable to expect that it is able to take into account the existence of a hydraulic connexion between cracks beyond a certain damage threshold. In order to take advantage of the analytical estimates based on Eshelby's problem, we adopt an ellipsoidal modelling of the fictitious porous inclusion associated with a given crack. The orientation of the ellipsoid is that of the crack, it is symmetric around the normal to the crack and has the same half-opening c and radius a as the crack itself. Its aspect ratio is $X = c/a \ll 1$. The self consistent estimate $\boldsymbol{k}^{SC}$ of the macroscopic permeability is the solution to :

$$\boldsymbol{k}^{SC} =< \boldsymbol{k}\cdot(\mathbf{1} + \boldsymbol{P}^{SC}\cdot(\boldsymbol{k} - \boldsymbol{k}^{SC}))^{-1} > \cdot < (\mathbf{1} + \boldsymbol{P}^{SC}\cdot(\boldsymbol{k} - \boldsymbol{k}^{SC}))^{-1} >^{-1} \tag{6}$$

where the tensor $\boldsymbol{P}^{SC}(\underline{z})$ is related to the Eshelby tensor $\boldsymbol{S}^{SC}(\underline{z})$ by $\boldsymbol{P}^{SC} = \boldsymbol{S}^{SC}\cdot\ \boldsymbol{k}^{SC^{-1}}$. The first order expansion of $\boldsymbol{S}^{SC}$ with respect to X reads :

$$(z\in\mathcal{C}^j) \quad \boldsymbol{S}^{SC} = \frac{1}{4}\pi X\mathbf{1} + (1 - \frac{3}{4}\pi X)\underline{n}^j\otimes\underline{n}^j \quad ; \quad (z\in\Omega^s) \quad \boldsymbol{S}^{SC} = \frac{1}{3}\mathbf{1} \tag{7}$$

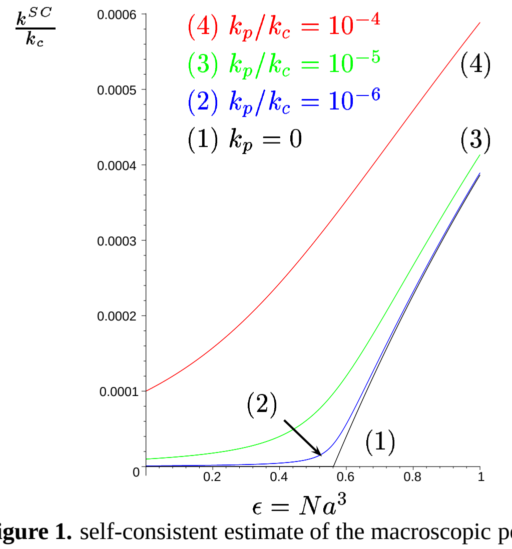

Figure 1. self-consistent estimate of the macroscopic permeability

We now consider the case of an isotropic distribution of crack orientations. $\boldsymbol{k}^{SC}$ is therefore isotropic : $\boldsymbol{k}^{SC} = k^{SC}\mathbf{1}$. In addition, the crack opening is taken uniform. Let N denote the crack density. The ratio k^{SC}/k_c determined from (6)-(7) is a function of the ratio k_p/k_c and of two geometrical characteristics, namely the "damage parameter" $\epsilon = Na^3$ and the aspect ratio X. For a constant value of X, the variations of k^{SC}/k_c versus ϵ for different values of k_p/k_c are plotted at figure 1. In the limit case $k_p = 0$, k^{SC} remains equal to 0 until a critical value $\epsilon^* = 9/16$. Beyond this threshold, the existence of a macroscopic permeability is the hydraulic counterpart for the appearance of a connected crack network. Hence, the self-consistent scheme is able to capture the concept of percolation threshold.
For $k_p \neq 0$, the variations of k^{SC} as a function of ϵ strongly depends on k_p/k_c. For higher values of this ratio, damage increases the macroscopic permeability without changing its order of magnitude. In contrast, figure 1 shows that k^{SC} can increase by several orders of magnitude when ϵ goes beyond ϵ^*, depending on the value of k_p/k_c, if this ratio is small enough. In this case, the variations of k^{SC} for $\epsilon < \epsilon^*$ are negligible ($k^{SC} \approx k_p$). For $\epsilon > \epsilon^*$, k^{SC} can be reasonably approximated by the estimate obtained for $k_p = 0$, for which an analytical expression is available :

$$k_p = 0 \;\Rightarrow\; k^{SC} = 3\pi k_c X \frac{16\epsilon - 9}{64\epsilon + 108} \tag{8}$$

Recalling that $k_c = c^2/3$, the above expression shows that $k^{SC} = \mathcal{O}(a^2 X^3)$. This result reveals the two contributions to the coupling between permeability and mechanical loading. On the one hand, the latter affects the crack opening and aspect ratio. On the other hand, it may induce crack propagation, i.e. an increase of a. Both phenomena can be addressed within a micromechanical framework. We hereafter briefly mention some results concerning crack propagation.

a micromechanical modelling of damage

For simplicity, we consider an isotropic crack propagation process in an elastic brittle solid (bulk modulus k^s, Poisson coefficient ν^s) induced by an isotropic loading (macroscopic stress $\boldsymbol{\Sigma} = \Sigma\mathbf{1}$ and strain $\boldsymbol{E} = E\mathbf{1}$). The pore space is made up of an isotropic distribution of cracks, connected and saturated by a fluid at pressure P. Let $\psi^*(E, P, \epsilon)$ denote the macroscopic potential energy of the cracked solid ([2],[3]). The thermodynamic force $\mathcal{F}$ associated with the damage parameter ϵ is shown to be equal to the derivative $-\partial\psi^*/\partial\epsilon$ ([4]). This means that the dissipation associated with the rates $\dot{E}$, $\dot{P}$ and $\dot{\epsilon}$ is equal to $-\dot{\epsilon}\,\partial\psi^*/\partial\epsilon$. We then adopt a Griffith type propagation criterion of the form $\mathcal{F} \leq \mathcal{F}_o$:

$$\mathcal{F} = k^s(E + \frac{p}{3k^s})^2 \mathcal{G}(\epsilon) \leq \mathcal{F}_o \quad \text{avec} \quad \mathcal{G}(\epsilon) = 648\frac{(1-\nu^2)(1-2\nu)}{(16\epsilon(1-\nu^2) + 9(1-2\nu))^2} \tag{9}$$

Once the propagation is initiated, the equality holds in (9). The evolution of ϵ can therefore be determined as a function of E and P. It is also found that the propagation is associated with a threshold of Terzaghi's effective stress :

$$\frac{8}{9k^s}\frac{1-\nu^2}{1-2\nu}(\Sigma + p)^2 \leq \mathcal{F}_o \tag{10}$$

References

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