

## A NEW SPECTRAL CLOSURE MODEL OF TURBULENT FLOWS WITH STABLE STRATIFICATION AND ITS APPLICATION TO ATMOSPHERIC SBLs

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**Summary** A new model for turbulent flows with stable stratification is presented. This model belongs in the class of the quasi-Gaussian closures; its parameters are calculated based upon a self-consistent recursive procedure of small-scale modes elimination starting at the Kolmogorov scale  $k_d$ . The model includes both vertical and horizontal eddy viscosities and diffusivities thus explicitly recognizing the anisotropy induced by stable stratification. There are significant differences in the behavior of these turbulent exchange coefficients with increasing stratification. Generally, the vertical coefficients are suppressed while their horizontal counterparts are enhanced. The model accounts for the combined effect of turbulence and internal waves on the exchange coefficients. A dispersion relation for internal waves in the presence of turbulence is derived. A threshold criterion for the wave generation in the presence of turbulent scrambling is obtained. The new model can be used to derive subgrid-scale parameterizations for LES and eddy viscosities and diffusivities for RANS models. The latter approach is used to develop a new  $K - \epsilon$  model which is tested in simulations of the atmospheric stable boundary layer (SBL) over sea ice. The new model performs well in both moderately and strongly stratified SBLs.

### THE SPECTRAL MODEL

The spectral closure theory is developed for a fully three-dimensional, incompressible, turbulent flow field with imposed homogeneous, vertical, stable temperature gradient; the flow is governed by the momentum, temperature and continuity equations in Boussinesq approximation,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \nabla) \mathbf{u} + \alpha g \theta \hat{e}_3 = \nu_0 \nabla^2 \mathbf{u} - \frac{1}{\rho} \nabla P + \mathbf{f}^0, \quad (1)$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{u} \nabla) \theta + \frac{d\Theta}{dz} u_3 = \kappa_0 \nabla^2 \theta, \quad (2)$$

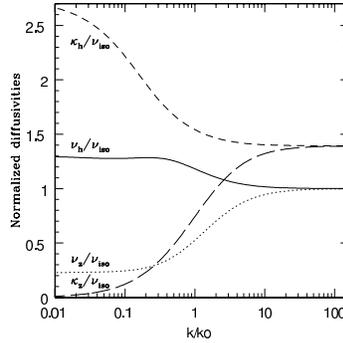
$$\nabla \cdot \mathbf{u} = 0, \quad (3)$$

where  $\mathbf{u}$  and  $\theta$  are the fluctuating velocity and the fluctuating potential temperature, respectively;  $P$  is the pressure,  $\rho$  is the constant reference density,  $\nu_0$  and  $\kappa_0$  are the molecular viscosity and diffusivity, respectively,  $\alpha$  is the thermal expansion coefficient,  $g$  is the acceleration due to gravity directed downwards,  $\frac{d\Theta}{dz}$  is the mean potential temperature gradient, and  $\mathbf{f}^0$  represents a large-scale external energy source customarily used in spectral theories of turbulence; it maintains turbulence in statistically steady state and may originate from large-scale shear instabilities. According to Kolmogorov theory of turbulence, the details of this forcing are immaterial in statistical description; its net effect is communicated to the fluid via a single integral parameter, the rate of the energy injection at large scales. Due to strong nonlinear interactions, the external forcing excites all Fourier modes down to the dissipative scale  $k_d$ . The modes exert indiscriminant, random agitation upon each other which manifests as a stochastic *modal* forcing  $\mathbf{f}$ . This forcing is used to replace the non-linear equations (1), (2) by modal stochastic equations

$$u_i(\mathbf{k}, \omega) = G_{ij}(\mathbf{k}, \omega) f_j(\mathbf{k}, \omega), \quad (4)$$

$$\theta(\mathbf{k}, \omega) = -\frac{d\Theta}{dz} u_3(\mathbf{k}, \omega) G_\theta(\mathbf{k}, \omega), \quad (5)$$

also known as the Langevin equations. Here,  $G_{ij}(\mathbf{k}, \omega)$  and  $G_\theta(\mathbf{k}, \omega)$  are the velocity and the temperature Green functions, respectively. They include terms accounting for the damping of a given mode by all other modes due to nonlinear interactions. Eventually, these terms are associated with  $k$ -dependent viscosities and diffusivities [1, 2]. The Langevin equations can be viewed as a device that facilitates the replacement of the original nonlinear Navier-Stokes and temperature equations by a system of linear, forced, stochastic equations in which the energy budget is systematically adjusted for every Fourier mode. In more rigorous interpretation, the replacement of the fully nonlinear Navier-Stokes equations by the Langevin equations represents a *mapping* of the original flow field onto a quasi-Gaussian field  $\mathbf{f}(\mathbf{k}, \omega)$  under the constraints of incompressibility and conservation of the modal energy flux. In the case of neutral stratification, this approach recovers some basic features of isotropic homogeneous turbulence including the Kolmogorov spectrum [1].



**Figure 1.** Normalized horizontal and vertical eddy viscosities and diffusivities as functions of  $k/k_O$ .

The parameters of the eddy damping are calculated using a systematic algorithm of successive averaging over small shells of velocity and temperature modes. The algorithm consists of computation of small increments to the vertical and the horizontal viscosities and diffusivities generated by the averaging over small shells of the velocity and the temperature modes using the Langevin equations (4) and (5). It results in a system of four coupled ODEs for turbulent viscosities and diffusivities,

$$\frac{d}{dk}(\nu_h, \nu_z, \kappa_h, \kappa_z) = -\frac{\epsilon}{k^5} R_{1,2,3,4}(\nu_h, \nu_z, \kappa_h, \kappa_z), \quad (6)$$

where  $\epsilon$  is the rate of the viscous dissipation;  $\nu_h$  and  $\nu_z$  are the horizontal and the vertical eddy viscosities, respectively;  $\kappa_h$  and  $\kappa_z$  are the horizontal and the vertical eddy diffusivities, respectively, and  $R_1$  through  $R_4$  are algebraic expressions. The procedure takes account of the combined effect of turbulence and internal waves. The computation starts at the Kolmogorov scale  $k_d$  where the initial values of the vertical and the horizontal viscosities and diffusivities are equal to their respective molecular values  $\nu_o$  and  $\kappa_0$  and is continued to an arbitrary wave number  $k < k_d$ . The system (6) can only be solved numerically. Solutions obtained for the non-dimensional variables  $\nu_h/\nu_{iso}$ ,  $\nu_z/\nu_{iso}$ ,  $\kappa_h/\nu_{iso}$  and  $\kappa_z/\nu_{iso}$  are presented in Fig. 1 as functions of the ratio  $k/k_O$ , where  $k_O = (N^3/\epsilon)^{1/2}$  is the Ozmidov wave number and  $\nu_{iso}$  is the eddy viscosity for neutral stratification ( $N = 0$ ) obtained with the same  $\epsilon$ .

### IMPLEMENTATION OF THE SPECTRAL RESULTS IN $K - \epsilon$ MODELING

The process of small scales elimination can be extended to the largest scales of the system, i.e., the integral length scale,  $k_L^{-1}$ . This approach is analogous to the Reynolds averaging and the resulting equations represent a sort of a RANS model. We have used  $\nu_z$  and  $\kappa_z$  to develop a  $K - \epsilon$  model based upon the spectral theory rather than the Reynolds stress closure. In simulations of SBLs, it was found necessary to generalize the formulation of the  $\epsilon$ -equation given in [3] to include the effect of stratification in addition to the rotation,

$$C_1 = C_1^0 + C_f Ro_*^{-1} - C_N Fr_*^{-1}, \quad (7)$$

where  $Ro_* = u_*/|f|L$ ,  $Fr_* = u_*/NL$ ,  $u_*$  is the friction velocity,  $f$  is the Coriolis parameter,  $C_1^0$  is the standard coefficient equal to 1.44,  $L = 0.16K^{3/2}/\epsilon$  is the turbulence macroscale used in the  $K - \epsilon$  modeling,  $C_f = 111$  and  $C_N = 0.58$  are empirical constants. The new  $K - \epsilon$  model has been tested in simulations of ABL over sea ice and compared with the data from Beaufort Arctic Storms Experiment (BASE) and LES [4]. The results of the simulations with the new  $K - \epsilon$  model are in good agreement with the LES for both cases of the moderate and strong stable stratification and provide significant improvement over the standard  $K - \epsilon$  model.

### References

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