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SELF-PROPULSION OF AN OSCILLATORY WING

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Summary In this paper we show that the oscillatory motion of an airfoil (wing) in an incompressible inviscid fluid can determine the apparition of a propulsive force. To this aim we discretize the lifting surface equation and calculate the pressure coefficients. Integrating, we calculate the average drag coefficient which for a range of values of the frequency may happen to be negative. The oscillatory motion of a wing can model the motion of fishes and birds. It is also of great importance in aeroelasticity.

THE INTEGRAL EQUATION

We consider a system of coordinates $Ox^{(1)}y^{(1)}z^{(1)}$ related to the wing and we introduce the dimensionless space coordinates $(x,z) = \left(\frac{x^{(1)}}{a}, \frac{z^{(1)}}{a}\right)$, taking the wing length a as reference length along the vertical direction ($Oz^{(1)}$-axis direction) and along the direction of the unperturbed uniform flow ($Ox^{(1)}$-axis direction). We introduce also the dimensionless space coordinate $y = \frac{y^{(1)}}{b}$, taking the wing half-span b as reference length along the $Oy^{(1)}$-axis direction. We denote by $D^{(1)}$ the wing projection on the $Ox^{(1)}y^{(1)}$ -plane. Let

$$0 = F(x^{(1)}, y^{(1)}, z^{(1)}) = z^{(1)} - h^{(1)}(x^{(1)}, y^{(1)}) \exp(i\omega t) \; ; \; (x^{(1)}, y^{(1)}) \in D^{(1)}, \tag{1}$$

$$\left|h^{(1)}\right| << 1, \; \left|\frac{\partial h^{(1)}}{\partial x^{(1)}}\right| << 1,$$

be the equation of the oscillating wing (we neglect the thickness of the wing).
In the sequel $\rho_0 Re(f \exp(i\omega t))$ is the jump of the pressure over the oscillating wing (ρ_0 is the density of the fluid at rest), ω is the frequency of the oscillation, V_0 is the translation velocity of the unperturbed flow with respect to the $Ox^{(1)}y^{(1)}z^{(1)}$ frame of refference, $\varpi = \frac{b}{a}$ is the aspect ratio.
We introduce the dimensionless functions and variables

$$h(x,y) = \frac{h^{(1)}(x^{(1)}, y^{(1)})}{a}, \; \tilde{\omega} = \frac{\omega a}{V_0}, \; \tilde{f}(x,y) = \frac{f(ax, by)}{V_0^2}, \; x_0 = x - \xi, \quad y_0 = y - \eta,$$

In the framework of the linearized theory the lifting surface equation for oscillatory airfoils is [1]:

$$\frac{\varpi}{4\pi} \int \int_D^* \tilde{f}(\xi,\eta) \exp(-i\tilde{\omega} x_0) \left(\int_{-\infty}^{x_0} \frac{\exp(i\tilde{\omega} u)}{(u^2 + \varpi^2 y_0^2)^{3/2}} du \right) d\xi d\eta =$$

$$= -\left(\frac{\partial h(x,y)}{\partial x} + i\tilde{\omega} h(x,y) \right), \; D = \{(x,y); (ax, by) \in D^{(1)}\}. \tag{2}$$

In the sequel we are going to put into evidence the singularities of the kernel. We have

$$\int_{-\infty}^{x_0} \frac{\exp(i\tilde{\omega} u)}{(u^2 + \varpi^2 y_0^2)^{3/2}} du = K^{(1)}(x,y;\xi,\eta) + K^{(2)}(x,y;\xi,\eta) + K^{(3)}(x,y;\xi,\eta),$$

where the kernel

$$K^{(1)}(x,y;\xi,\eta) = \frac{1}{\varpi^2 y_0^2} \left(1 + \frac{x_0}{\sqrt{x_0^2 + \varpi^2 y_0^2}} \right),$$

has a strong singularity, the kernel

$$K^{(2)}(x,y;\xi,\eta) = -\frac{i\tilde{\omega}}{\sqrt{x_0^2 + \varpi^2 y_0^2}} + \frac{\tilde{\omega}^2}{2} \ln\left(-x_0 + \sqrt{x_0^2 + \varpi^2 y_0^2} \right),$$

has weak singularities and the kernel

$$K^{(3)}(x,y;\xi,\eta)=\frac{\widetilde{\omega}}{\varpi\left|y_0\right|}\left(K_1\left(\widetilde{\omega}\varpi\left|y_0\right|\right)-\frac{1}{\varpi^2y_0^2}-\frac{\widetilde{\omega}^2}{2}\ln\frac{\widetilde{\omega}\varpi\left|y_0\right|}{2}\right)+$$
$$+i\frac{\pi}{2}\frac{\widetilde{\omega}}{\varpi\left|y_0\right|}\left(I_1\left(\widetilde{\omega}\varpi\left|y_0\right|\right)-L_{-1}\left(\widetilde{\omega}\varpi\left|y_0\right|\right)+\frac{2}{\pi}\right)+\frac{\widetilde{\omega}^2x_0}{2\sqrt{x_0^2+\varpi^2y_0^2}}+\frac{\widetilde{\omega}^2}{2}\ln\frac{\varpi}{2}+$$
$$+\int_0^{x_0}\frac{\exp(i\widetilde{\omega}u)-1-i\widetilde{\omega}u+\frac{1}{2}\widetilde{\omega}^2u^2}{(u^2+\varpi^2y_0^2)^{3/2}}du,$$

has no singularity. (I_1 and K_1 are Bessel functions and L_{-1} is a Strouve function.)

THE PROPULSIVE FORCE FOR A DELTA WING

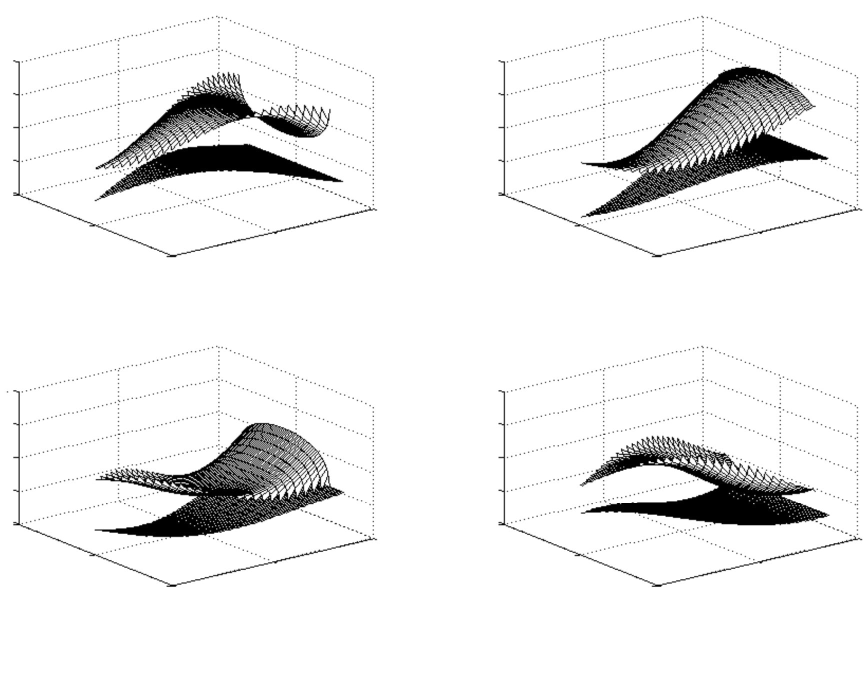
Employing for each type of kernel adequate quadrature formulas, we discretize (2) obtaining the values of $\tilde{f}$ in the nodes of the grid. With the the formulas

$$C_p(x^{(1)},y^{(1)},t)=Re[\tilde{f}(x,y)\exp(i\omega t)],\ C_D(t)=-2\int\int_D n_xC_p(ax,by,t)dxdy,\widetilde{C}_D=\frac{1}{T}\int_0^T C_D\left(t\right)dt$$

where T is the period of the oscillation, we calculate the pressure coefficient field, the drag coefficient and the average drag coefficient. Considering the oscillating delta wing whose equation is

$$h(x,y)=\alpha\exp(i\widetilde{\omega}_1x),\qquad(x,y)\in D=\left\{(x,y)\,;0<y<\left|x\right|,0<x<1\right\},$$

we calculate the average drag coefficient and we notice that if $\tilde{\omega}$ surpasses a certain critical value, the average drag coefficient becomes negative i.e. there appears a *propulsive force*. In the following figure we present the oscillating wing (below) and the pressure coefficient fields (above) for the values of the nondimensional time $\frac{V_0}{a}t\in\{1,1,2,4\}$ and for the nondimensional frequencies $\tilde{\omega}=\pi/2,\ \widetilde{\omega}_1=6\pi/5$. We considered $\varpi=1/8$.



References

[1] D. Homentcovschi, Theory of the lifting surface in unsteady motion in an inviscid fluid, *Acta Mechanica* **27**, 205-216, 1977.