MODELLING OF ELASTIC-PLASTIC OR VISCOPLASTIC MATERIALS SENSITIVE TO THE TYPE OF THE PROCESES – DIFFERENT APPROACHES

Anguel I. Baltov[*], Georgi B. Stoychev[**]
[*]*Bulgarian Academy of Sciences, Institute of Mechanics, BG 1113 Sofia, Bulgaria*
[**]*Technical University Sofia, BG 1000 Sofia, Bulgaria*

Summary Modelling of the nonelastic materials sensitive to the type of the processes are presented. Two different approaches are given. Some experimental verifications and model parameter identifications are demonstrated for different characteristic models.

Introduction

Sensitivity to the type of the process is universal property well known in many practical cases. The presentation of this property is like different behavior of the nonelastic material during fore example: extension or compression; loading or unloading; hardening or softening; plastic deformation with athermal or thermofluctuational micromechanismes; relaxation or creep; etc. The principle of the modeling of this type of material behavior is based on the introducing models with different material constants according to the type of process in consideration.

In this paper we will give two approaches:

(1) First: It is more or less classical approach, using nonsymmetric yield surfaces in the stress space;
(2) Second: It is on the base of extended strain space, introducing the process type indicators

and incremental constitutive relations with different material functions for different process types.

First approach

Grey cast iron, some steels, light alloys, ceramics, polymers, composites, rocks and soils etc., show in tests different plastic behavior and strength in tension and compression, volumetric plastic deformation and other nonclassical effects. A nonclassical effect for this material is the volumetric dilatation in tension, compression and torsion. The classical theory of plasticity which is based on the Huber-von Mises-Henckly or the Tresca conditions in the case of significant nonclassical effects is unworkable. In the case of different behavior in tension and compression we propose extension of the classical yield conditions formulated on the base of the modified plastic work as a hardening parameter.

We propose the yield function

$$f = \sqrt{J_2} + \varphi(\chi)\sigma_1 - \psi(\chi) = 0\,,$$

where J_2 is the second invariant of the stress deviator, σ_1 is the maximum principal stress, χ is the hardening parameter, $\varphi(\chi)$ and $\psi(\chi)$ are material functions determined by uniaxial tests

$$\varphi(\chi) = \frac{\sigma_C(\chi) - \sigma_T(\chi)}{\sqrt{3}\sigma_T(\chi)}\,,\quad \psi(\chi) = \frac{\sigma_C(\chi)}{\sqrt{3}}\,.$$

The functions $\sigma_T(\chi)$ and $\sigma_C(\chi)$ can be obtained as a current yield stresses in uniaxial tension and compression respectively.

The hardening parameter taking in account the pressure sensitivity of the grey cast iron can be defined as

$$\chi = \int \frac{1}{\omega(\sigma_m,\theta)}\sigma_{ij} d\varepsilon_{ij}^p\,,\ \text{with}\ \omega(\sigma_m,\theta) = \frac{b_1 - b_2}{1 + \exp\left(\dfrac{\sigma_m - b_3}{b_4}\right)} + b_2$$

where σ_{ij} is the stress tensor and $d\varepsilon_{ij}^p$ is the increment of the plastic strain tensor, $\sigma_m = I_1/3$ is the mean stress and b_1, b_2, b_3, b_4 are material parameters depending on the loading type. These parameters can be identified by simple tests in tension, compression and torsion.

Since the experimental results indicate plastic volume dilatation in tension, compression and pure torsion calculation of transverse strain using associated flow rule gives unrealistic values independent of the choice of the yield function. More accurate modeling of the volume change in the elastoplastic region can be obtained using non-associated flow rule. The plastic potential is proposed in the following form

$$g = J_2 + \gamma(\chi) I_1 + \vartheta(\chi) I_1^2\,,$$

where $\gamma(\chi)$ and $\vartheta(\chi)$ are functions of the hardening parameter and can be identified using experimental data from uniaxial tests in tension and compression. The increment of the volumetric plastic deformation can be determined from the flow rule $d\varepsilon_V^p = d\lambda \delta_{ij} \dfrac{\partial g}{\partial \sigma_{ij}}$, where $d\lambda$ is plastic multiplier. The stress increment can be computed via the elastic stress-strain relations $d\sigma_{ij} = D_{ijkl} d\varepsilon_{kl}$ with $d\varepsilon_{ij}^e = d\varepsilon_{ij} - d\varepsilon_{ij}^p$ since the total strain increment can be divided to an elastic and plastic part. Based on the plastic consistency condition the plastic multiplier can be expressed in the form

$$d\lambda = \omega(\sigma_m, \theta) \frac{\dfrac{\partial f}{\partial \sigma_{kl}} D_{klmn} d\varepsilon_{mn}}{-\dfrac{\partial f}{\partial \chi}\sigma_{rs}\dfrac{\partial g}{\partial \sigma_{rs}} + \dfrac{\partial f}{\partial \sigma_{kl}} D_{klmn} \dfrac{\partial g}{\partial \sigma_{mn}}},$$

where D_{klmn} is fourth order tensor of elastic material constants and $d\varepsilon_{mn}$ is the tensor of total strains.
The verification of the model was realized using experimental data for grey cast iron. Experimental tensile stress-strain relation $(\sigma_1 - \varepsilon_1)$ was used to obtain the function $\sigma_T(\chi)$. There is a good agreement between the computed values and the experimental data from uniaxial and biaxial tests with tubular specimens.

Second approach

For modeling the material's sensitivity to the type of the process we introduce: (1) six dimensional strain vector $\in_\alpha$, (α=1, 2… 6) on the base of the strain tensor; (2) Back stress vector $\sum_\alpha^\times$ on the base of the back stress tensor, (3) Damage vector D_α on the base of the damage tensor; (4) Temperature T; (5) internal time τ. We build a linear seven dimensional vector space $L = Y_6 \times I_T \times I_T^+$, where Y_6 is the six dimensional vector space, I_T is the one dimensional axis for T and I_T^+ is the one dimensional positive axis for τ.

The process measures in the fixed point (x_i) and the time t are $\Lambda_t \equiv \{\in_\alpha, \Sigma_\alpha^a, D_\alpha, T, \tau\}$, with $\Sigma_\alpha^a = \Sigma_\alpha - \Sigma_\alpha^\times$, $\in_\alpha = \in_\alpha^{(rev)} + \in_\alpha^{(irrev)}$.

The process duration is from t_o to t_f e.g. $t \in [t_o, t_f]$. At the fixed time $t = t_h$ we assume to know the state measures $\Lambda_h \equiv \{\in_{\alpha(h)}, \Sigma_{\alpha(h)}^a, D_{\alpha(h)}, T_h, \tau_h\}$. This state is reached from the process in previous time interval Δt with the type according to the values of the indicator manifold $\Lambda_c \equiv \{g, d, q\}$. All indicators take the values: (-1) – for the small quantities: (0) – for the moderate quantities and (+) – for the big quantities. g is for the strain-rate range; d is for the damage range; q is for temperature range.
We examine the process evolution during the actual process time interval $\Delta t_h = t_a - t_h$, where t_a is the actual process time. All process measures change $\Delta\Lambda_h \equiv \{\Delta\in_\alpha, \Delta\Sigma_\alpha^a, \Delta D_\alpha, \Delta T, \Delta\tau\}$. The type of the process during this small evaluation is identify using the fallowing indicator manifold $\Lambda_d \equiv \{\lambda_E, \lambda_D, \theta, \upsilon\}$. λ_E gives the type of the deformational process development; λ_D gives the type of the damage process development;

$$\theta = \frac{\Delta T}{|\Delta T|} = \begin{cases} +1 \text{ for warming;} \\ 0 \text{ for isotehermal process change;} \\ \text{-1 for cooling.} \end{cases}$$

$$\upsilon = \frac{\Delta\tau}{|\Delta\tau|} = \begin{cases} 0 \text{ for reversibal process change;} \\ +1 \text{ for irreversible process change.} \end{cases}$$

(-1) is impossible according to thetthermodynamical restrictions.
The constitutive equations are in incremental form:

$$\Delta\Sigma_\alpha = E_{\alpha\beta}\Delta \in_\beta^{(rev)} + N_{\alpha\beta}\Delta \in_\beta^{(irrev)};$$
$$\Delta\Sigma_\alpha^\times = R_{\alpha\beta}\Delta \in_\beta^{(irrev)};$$
$$\Delta D_\alpha = Q_{\alpha\beta}\Delta \in_\beta^{(irrev)},$$

where the matrixes ($E_{\alpha\beta}$), ($N_{\alpha\beta}$), ($R_{\alpha\beta}$) and ($Q_{\alpha\beta}$) depend on the state Λ_h at the moment t_h and on the two indicator groups: Λ_c and Λ_d. It is possible to define the internal time in connection with the dissipation energy e.g.

$$\tau = \int_{t_0}^{t_\lambda} \Sigma_\alpha^a . \Delta \in_\alpha^{(irrev)} \, dt .$$

We give, like examples, some models for different particular cases (deformational type sensitive material; sensitive to type of rheological process; sensitive to the type of loading etc). We give also some experimental verifications and model parameter identifications for various materials – some composites, metals, woods etc.