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SPONTANEOUS SIGN REVERSALS IN SELF-ORGANIZED STATES OF FORCED TWO-DIMENSIONAL TURBULENCE ON A BOUNDED SQUARE DOMAIN

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Summary The inverse energy cascade present in two-dimensional (2D) turbulence leads to the formation of large-scale flow structures. In the case of decaying 2D turbulence on a square domain with no-slip walls the flow usually shows self-organization into a single domain-filling circulation cell with an associated increase in the total angular momentum of the flow — a process referred to as 'spontaneous spin-up'. Subsequently, this organized state may persist until all energy is depleted by viscous dissipation and the fluid eventually comes to rest. In contrast, if the energy of the flow is maintained by some external forcing mechanism, a spectacularly different behaviour may be observed. Boundary layers present at the domain walls can destabilize the organized state, such that the dominating circulation cell collapses, and the self-organization process may start anew. Most strikingly, the circulation may even show sign reversal. This flow behaviour has been investigated by high-resolution numerical simulations based on spectral techniques.

DECAYING 2D TURBULENCE

A well-known feature of two-dimensional (2D) turbulence is the inverse energy cascade, according to which energy is passed from small to large scales of motion. It is generally assumed that the inverse cascade is linked to the formation of vortex structures. In the case of slowly decaying turbulence, these structures show complicated interactions, resulting in a decreasing number of vortices of increasing size. Numerical simulations carried out for a turbulent flow on a double periodic square domain have revealed that the flow eventually becomes organized in the form of a combination of two cells of positive and negative circulations [1]. In contrast, the 'final state' of decaying 2D turbulence on a square domain with no-slip boundaries consists of a large central cell with either positive or negative circulation, surrounded by a shielding ring of negative or positive vorticity, respectively, such that the total circulation of the flow is zero (as dictated by the no-slip condition at the domain boundaries). This long-time behaviour has been found both in laboratory experiments and in high-resolution numerical flow simulations. A remarkable observation was that in many cases the total angular momentum $L(t)$ of the flow, which is initialized randomly, with $L(t=0) \simeq 0$ shows a sudden change to non-zero values — a feature termed 'spontaneous spin-up' [2]. This total spin-up of the fluid is directly associated with the self-organization of the flow into a single larger vortex structure that fills the domain almost completely. In the next stage of the flow evolution, the absolute angular momentum $|L(t)|$ shows a very slow decay to zero for very late times. It is important to note that the no-slip boundary condition is a prerequisite for the spin-up, as the angular momentum $L(t)$ is an irrelevant quantity for the flow evolution on a double-periodic domain. Also, the square domain geometry is important, spin-up being virtually absent on a circular domain [3] or on a long rectangular domain in which the 'final state' consists of a linear (domain-filling) array of counter-rotating cells [4]. Obviously, the change of the total angular momentum during the spontaneous spin-up is connected with the action of forces at the domain boundaries. Numerical simulations have revealed that — for the case of a square geometry — the contribution of the inviscid normal stress (i.e., the pressure) is much larger than the effects of viscous shear and normal stresses.

FORCED 2D TURBULENCE

In recent direct numerical simulations of stochastically forced 2D turbulent flow on a square domain with no-slip boundary conditions we also observed spontaneous spin-up behaviour, although remarkably different from the decaying case. In these forced turbulence experiments one observes several consecutive events of rapid increase and decrease of $|L(t)|$, often with sign reversal of $L(t)$ between neighbouring peaks in $|L(t)|$.

The evolution of the flow is governed by the vorticity equation

$$\frac{\partial \omega}{\partial t} + (\mathbf{v} \cdot \nabla)\omega = \nu \nabla^2 \omega + q \tag{1}$$

with $\mathbf{v}(\mathbf{r},t) = (u,v)$ the velocity vector and $\mathbf{r} = (x,y)$ the position vector on a square domain $\mathcal{D}$ defined by $\{-1 \leq x \leq 1; -1 \leq y \leq 1\}$; furthermore, $\omega(\mathbf{r},t) \equiv \hat{\mathbf{z}} \cdot \nabla \times \mathbf{v} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the vorticity component perpendicular to the flow field, ν is the kinematic viscosity, and q represents the stochastic external forcing of the flow. Subject to the no-slip condition $\mathbf{v} = 0$ on domain boundary $\partial\mathcal{D}$, and initially, $\omega(\mathbf{r},0) = 0$ on $\mathcal{D}$, equation (1) is solved numerically using a pseudo-spectral code based on Chebyshev expansions and a semi-implicit Adams-Bashforth Crank-Nicolson time integration scheme [5]. The forcing q is modelled as a first order Markov process [6]. The total angular momentum $L(t)$ of the flow with respect to the domain centre is defined as

$$L(t) = \int_{\mathcal{D}} (\mathbf{r} \times \mathbf{v}) \cdot \hat{\mathbf{z}} dA \; . \tag{2}$$

A typical example of the evolution of the normalised angular momentum $L'(t) = L(t)/L_u(t)$ — where $L_u(t)$ is the angular momentum associated with a uniform rotation with kinetic energy $E(t)$ — during a simulation with integral scale

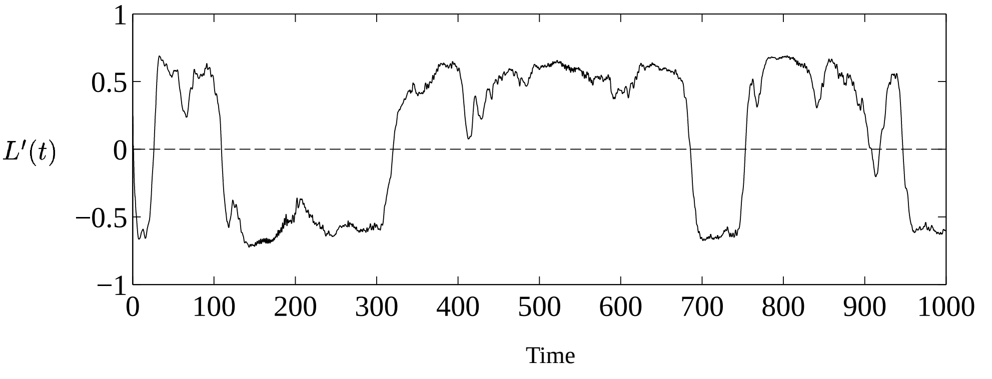

Figure 1. *Evolution of the normalized angular momentum $L'(t)$, showing distinct phases of spin-up.*

Reynolds number $Re = (Ud/\nu) \simeq 3000$ (where U is the root-mean-square velocity and d the domain size) is shown in Figure 1. It is clearly seen that — although being equal to zero initially — the angular momentum tends to reach a non-zero value, i.e. the flow exhibits spontaneous spin-up. Moreover, several sign changes in L' are observed to occur: apparently the flow reverses abruptly in those cases.

In fact, a few different stages can be distinguished in the flow evolution: first the flow is showing evidence of self-organization or spin-up, as can be observed from the build-up of a larger circulation cell and the associated angular momentum, followed by a relatively rapid destruction of the cell and a dramatic decrease of the flow's total angular momentum. Subsequently, the flow becomes organized again into a larger circulation cell — either with the same or with opposite rotation direction as the previous cell. The no-slip boundaries enclosing the flow domain play a crucial role in the repeated destruction of the organized flow state. The viscous layers at the walls contain oppositely-signed vorticity, as a consequence of the no-slip condition. Once the larger cell is established, the boundary layers detach, giving rise to the formation of smaller vortices in the corners of the domain. The corner vortices gradually grow in size and strength, and subsequently start to interact with the flow in the interior. The central cell is thus progressively eroded, it soon becomes unstable, and then breaks down rapidly. At that stage, the flow has become irregular, consisting of filamentary structures without any overall coherence. Subsequently the self-organization process may start anew, possibly leading to a flow cell with oppositely signed circulation.

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