ARBITRARILY WIDE-ANGLE WAVE EQUATIONS AND THEIR APPLICATIONS TO UNBOUNDED DOMAIN MODELING AND SUBSURFACE IMAGING

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INTRODUCTION

Standard (full) wave equations are used to model the propagation of disturbances in acoustic and elastic media. These disturbances tend to propagate in all directions, i.e. 360° range of angles. In contrast to full wave equations, one-way wave equations (OWWEs), as their name implies, represent waves propagating in 180° range of angles. Due to this special property, OWWEs have found applicability in the areas of unbounded domain modeling, underwater acoustics and subsurface imaging. In order to make OWWEs computationally tractable, they are approximated. While there are many good approximations of OWWE are available, most of them have limitations with respect to their applicability, especially for more complex problems involving wave propagation in anisotropic elastic and poro-elastic media. Most of the approaches to date are based on rational approximation of the factorized wave equation. The fundamental difficulty with the existing approximations of OWWE for complex anisotropic media is that there do not appear to be a plausible way of factorizing the wave equation.

With the goal of developing OWWE for complex media, Guddati [1] has recently developed a systematic framework for deriving OWWEs that are of arbitrarily high accuracy. Named Arbitrarily Wide Angle Wave Equations (AWWE), they do not require explicit factorization and, depending on the order of the approximation, tend to accurately propagate waves in a range of angles as high as 180°. This presentation includes a brief overview of the procedure, followed by the application of AWWE to subsurface imaging and modeling acoustic waves in unbounded domains.

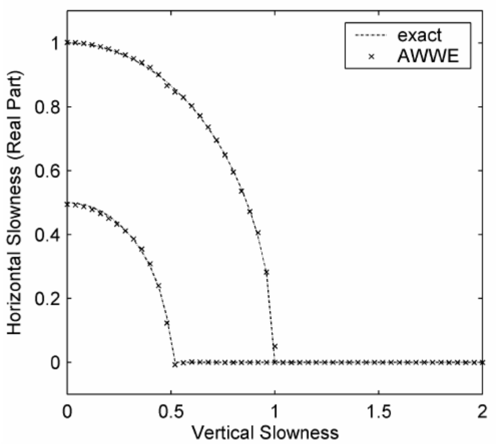
ARBITRARILY WIDE ANGLE WAVE EQUATIONS

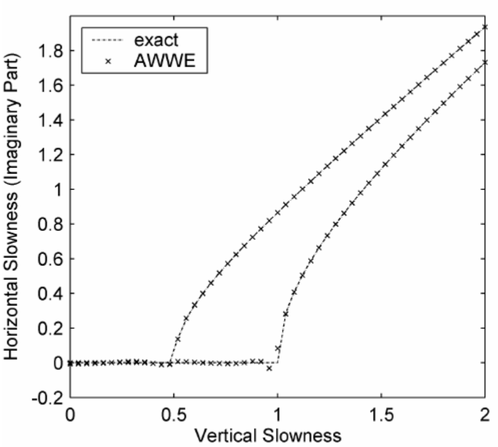
AWWEs are derived using a five-step procedure. The first step uses the idea of representing one-way wave equations in terms of half-space stiffness, subsequently reducing the problem into approximating the half-space stiffness. The half-space is discretized using finite element mesh (step 2), and the resulting discretization error in half-space stiffness is completely eliminated with the help of special integration rules (step 3). Fourth step involves truncating the number of elements to make it computationally tractable, while the final step requires the use of imaginary finite-element lengths to represent traveling waves, or complex lengths to represent both traveling and evanescent waves. The resulting AWWE would take the form, for example for elastic waves,

$$\begin{Bmatrix} \mathbf{G}_{xx}\dfrac{\partial^2 \mathbf{u}}{\partial x^2} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{Bmatrix} + \left(\mathbf{\Lambda}_1+\mathbf{\Lambda}_2\right)\frac{\partial^2}{\partial t^2}\begin{Bmatrix} \mathbf{u} \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{n-1} \end{Bmatrix} + \mathbf{\Lambda}_3\frac{\partial^2}{\partial z \partial t}\begin{Bmatrix} \mathbf{u} \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{n-1} \end{Bmatrix}$$

$$+\frac{\partial}{\partial z}\left(\mathbf{\Lambda}_3^T\frac{\partial}{\partial t}\begin{Bmatrix} \mathbf{u} \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{n-1} \end{Bmatrix}\right) - \frac{\partial}{\partial z}\left(\mathbf{\Lambda}_4\frac{\partial}{\partial z}\begin{Bmatrix} \mathbf{u} \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{n-1} \end{Bmatrix}\right) = \mathbf{0}.$$

In the above, $\mathbf{u}$ is the displacement vector, while $\mathbf{u}_1,\cdots,\mathbf{u}_{n-1}$ are auxiliary variables defined implicitly by the equation. $\mathbf{G}_{xx},\mathbf{\Lambda}_1,\mathbf{\Lambda}_2,\mathbf{\Lambda}_3,\mathbf{\Lambda}_4$ are positive definite or positive semi-definite coefficient matrices. The accuracy of AWWE depends on the number of auxiliary variables as well as AWWE parameters (reference phase velocities) that determine the coefficient matrices. The accuracy of AWWE is clearly illustrated in the adjacent figure, which contains the slowness diagram for the elastic AWWE (using six auxiliary variables) with pressure wave velocity $\alpha = 2$ and shear wave velocity $\beta = 1$. The exact slowness is also shown in the same figure. It is clear from the figure that AWWE was able to approximate the slowness in a highly accurate manner. Similar observations were made for anisotropic acoustic and elastic media.





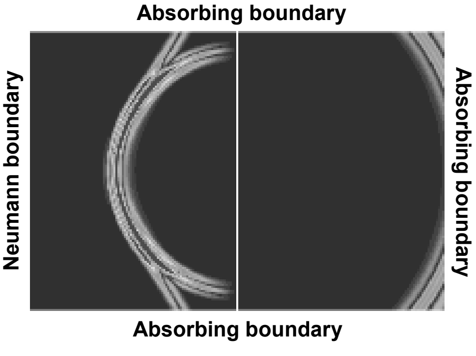
Salient Features of AWWE

AWWEs have several desirable properties: (a) AWWEs represent forward propagating waves associated with the general wave equations, even in anisotropic and poro-elastic media. The only restriction of the procedure is that the governing differential equation (full wave equation) must be second order. (b) AWWEs are exact for waves propagating with phase velocities coinciding with the reference phase velocities. Thus, knowledge of the approximate propagation characteristics could be used to easily design an effective AWWE. (c) AWWEs can not only represent propagating waves, but also evanescent waves. This is done by simply choosing imaginary or complex-valued reference phase velocities. (d) Increasing the number of auxiliary variables will make the AWWE accurate for wider angles of propagation. (e) The choice of complex reference phase velocities are expected to aid in developing stable approximations in cases where one-way approximations could be unstable (e.g. elastic wave propagation).

APPLICATION TO ACOUSTIC IMAGING

This section contains a representative result from an AWWE-based imaging procedure [2] applicable for acoustics waves. The problem involves detecting a hidden crack by sending ultrasonic waves into the structure and measuring the reflected waves at the surface. In order to test the imaging algorithm, the surface trace is generated synthetically using a computational experiment with the help of forward finite-difference wave propagation model. The resulting surface response is processed using a downward continuation procedure based on acoustic AWWE to predict the location of the crack. Adjacent figure contains the actual crack (bold white line), as well as the predicted crack location. As explained in reference [2], the accuracy of this prediction is significantly better than using more conventional 15° and 45° wave equations that are popular in seismic migration literature.

APPLICATION TO UNBOUNDED DOMAIN MODELING



Unbounded domain modeling using AWWE can be shown to be equivalent to the continued fraction absorbing boundary conditions (CFABC) previously developed in [3]. AWWE formalism, however, helps in extending CFABC to elastic media, media with corners and heterogeneous media. The adjacent figure is a representative result from reference [4], which shows the wave front snapshot for an explosion. The domain consists of a vertical acoustic layer attached to a half space extending to the right. The explosion is at the middle of the interface. It can be clearly seen that all the waves are being absorbed by the absorbing boundaries in a very effective manner.

CONCLUSIONS

Arbitrarily wide-angle wave equations (AWWEs) are highly accurate approximations of OWWE. Since they do not need explicit factorization, they are applicable for simulating one-way propagating complex heterogeneous and anisotropic, elastic and poro-elastic media. Preliminary applications of AWWE to acoustic imaging and unbounded domain modeling show excellent promise of AWWE. Ongoing research focuses on the application of AWWE to wider array of problems including elastic imaging and modeling unbounded elastic and poro-elastic domains.

Acknowledgement

This material is based upon work supported by the National Science Foundation under Grant No. CMS-0100188. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

[1] Guddati M.N.: Arbitrarily Wide Angle Wave Equations for Complex Media, to be submitted.
[2] Guddati M.N., Heidari A.H: Acoustic Migration with Arbitrarily Wide Angle Wave Equations, *Geophysics*, in review.
[3] Guddati M.N., Tassoulas J.L.: Continued Fraction Absorbing Boundary Conditions for the Wave Equation, Journal of Computational Acoustics, **8**: 139-156, 2000.
[3] Guddati M.N., Lim K.W.: Continued-Fraction Absorbing Boundary Conditions for Corner Regions, to be submitted.