FERROHYDRODYNAMIC HELE-SHAW CELL FLOWS AND INSTABILITIES WITH SIMULTANEOUS DC AXIAL AND IN-PLANE ROTATING MAGNETIC FIELDS

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EXPERIMENTS

New flows and instabilities are presented for a ferrofluid drop contained in glass Hele-Shaw cells of 0.9 - 1.4 mm gap with simultaneously applied in-plane rotating and DC axial uniform magnetic fields. The rotating uniform magnetic field is formed using a stator winding from a 2 pole induction motor which is excited by balanced three phase currents. The fluoro-carbon based ferrofluid has saturation magnetization ≈400 Gauss and low field magnetic susceptibility of $\chi \approx 3$. The ferrofluid is surrounded by a 50/50 mixture of isopropyl alcohol and deionized water which prevents ferrofluid wetting of the glass plates. The rotational field strength is up to 100 Gauss rms at frequencies 20-40 Hz while a DC axial field is varied from 0 to ~250 Gauss [1].

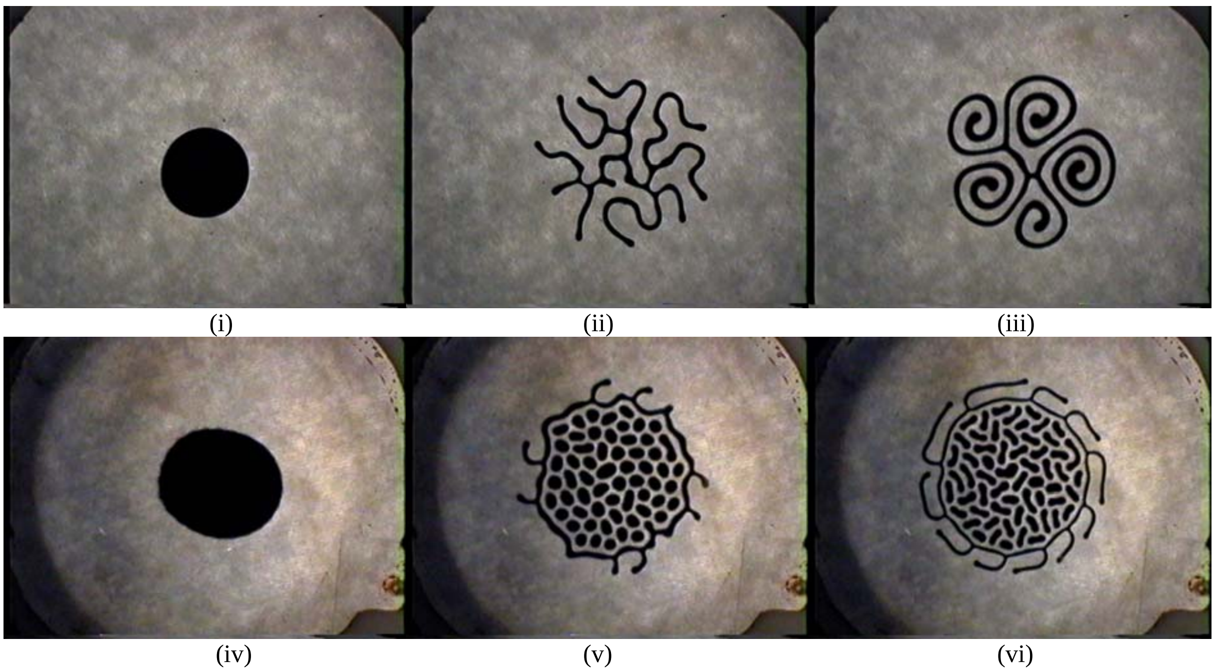


Figure 1 – Spiral pattern formed when a 100 microliter ferrofluid droplet in a 1.1 mm gap Hele-Shaw cell (i) is stressed by a 155 Gauss uniform DC axial magnetic field to form the labyrinth in (ii) and then an in-plane uniform clockwise rotating magnetic field of 47.5 Gauss rms at 20 Hz is applied to create a spiral flow (iii). A larger 200 microliter drop (iv) stressed by a 40 Gauss rms, 25 Hz uniform rotating field, is held together without a labyrinth until the uniform DC axial magnetic field is about 186 Gauss, subsequently forming the discrete droplet structure in (v). As the DC field is further increased to 228 Gauss the drops and "surface hair" further deform.

The representative images in Figure 1 describe two experiments with a 1.1 mm Hele-Shaw cell gap. The first experiment, images (i)-(iii), uses 100 microliters of ferrofluid and the DC axial field is first increased to 155 Gauss and then the clockwise rotating field at 47.5 Gauss rms, 20 Hz is turned on. Figure (i) shows the circular ferrofluid drop before the magnetic field is applied, while figure (ii) shows the labyrinth pattern formed by the ferrofluid after only the DC axial field has been applied. Then the clockwise rotating field is applied and the spikes begin to curl in on themselves, eventually forming the smooth spiral pattern shown in figure (iii). The smooth spirals form from viscous shear as the clockwise rotating magnetic field causes clockwise flow on the outside ferrofluid surfaces which return on the inside surfaces.

The second experiment, with 200 micro-liters of ferrofluid, is seen in figures (iv)-(vi). The test cell is first placed in the clockwise rotating field which causes a clockwise flow that holds the fluid drop together without spikes. Then a DC axial field is gradually applied. This results in the ferrofluid drop appearing to expand before the phase-like transition to figure (v) at a critical DC magnetic field strength around 186 Gauss. Careful observations show that the pattern of figure (v) forms at ~170 Gauss DC field under a thin ferrofluid coating on the top glass plate, which then abruptly peels away at slightly increased DC axial magnetic field. In figure (vi) the magnetic field is increased to 228 Gauss.

PRELIMINARY MODEL

A minimum free-energy analysis, like that used in modeling ferrofluid labyrinth phenomena [2], is performed to model the phase transformation from a single large drop to many discrete small drops. The model assumes that the initial large cylindrical drop has a radius *R* and height *t*, equal to the Hele-Shaw gap thickness, which at a critical DC axial magnetic field strength spontaneously form *N* smaller droplets of radius *r* and height *t*, where to conserve mass, $R = \sqrt{N}r$. The uniform applied DC axial field $\bar{H}_0$ magnetizes the ferrofluid drop and droplets to magnetization $\bar{M}$ generating a demagnetizing field $\bar{H}_d = -\bar{M}D$ where

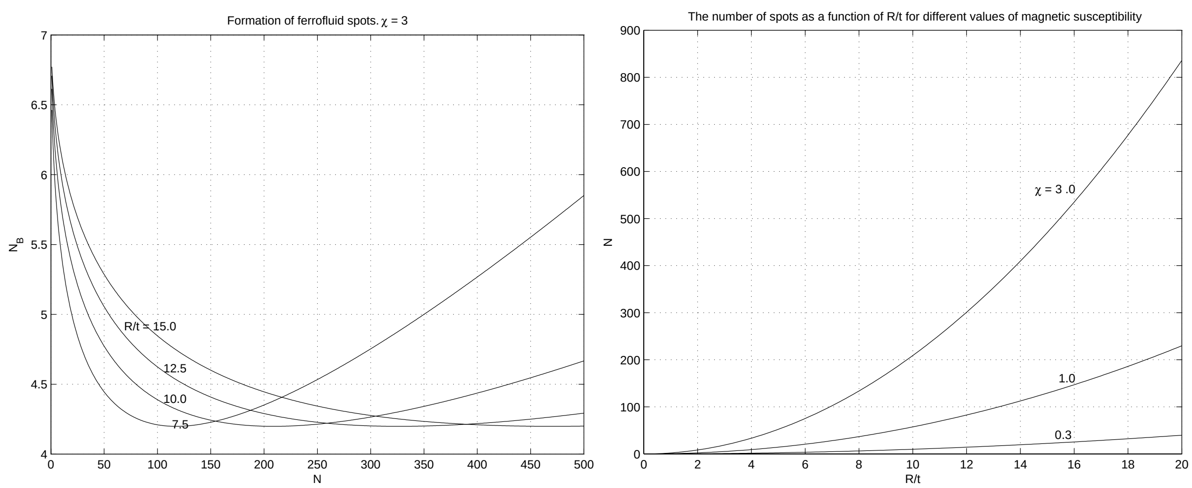
$$D \approx 1 - \frac{1}{\sqrt{\left(\frac{2r}{t}\right)^2 + 1}} = 1 - \frac{1}{\sqrt{\frac{1}{N}\left(\frac{2R}{t}\right)^2 + 1}} \tag{1}$$

is the approximate demagnetization coefficient for each droplet of radius *r*. The magnetization for linearly magnetizable ferrofluid with magnetic susceptibility χ is then

$$\bar{M} = \chi \bar{H} = \chi \bar{H}_0 / (1 + \chi D) \,. \tag{2}$$

The energy in the *N* droplets, each with surface tension γ, is assumed to be due to interfacial surface energy, $U_S = N 2\pi r t \gamma = 2\pi R t \gamma \sqrt{N}$ and to magnetization energy [3]

$$U_M = -\frac{\mu_0}{2}\int \bar{M} \bullet \bar{H}_0 dV = -\frac{\mu_0}{2}\frac{N\pi r^2 t \chi H_0^2}{1+\chi D} = -\frac{\mu_0}{2}\frac{\pi R^2 t \chi H_0^2}{1+\chi D} \,. \tag{3}$$



(a) (b)

Figure 2 – Calculated minimum energy condition of (a) magnetic Bond number $N_B = \mu_0 H_0^2 t/(2\gamma)$ versus number of phase transformation droplets *N* for various values of *R/t* with magnetic susceptibility $\chi = 3$; (b) The number of droplets *N* at minimum N_B as a function of *R/t* for various values of χ. For our experiments with a 200 microliter drop, shown in Fig. 1 (iv)-(vi), *R/t*≈7.

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References

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