INVERSE MAGNUS FORCE IN FREE MOLECULAR FLOW

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Summary We present a particle dynamics method for calculating the transverse force on a spinning solid translating in a rarefied gas and show that it is in the opposite direction to the classical Magnus effect in continuum flow. For a surprisingly large class of shapes, this inverse Magnus force is steady and given by a universal expression.

EXTENDED SUMMARY

The earliest known description and explanation of the transverse deflection of a spinning projectile is contained in Newton's first scientific publication, "New theory about light and colors" [1]. Newton "had often seen a Tennis ball, struck with an oblique Racket, describe such a curved line" and wondered if particles of light are similarly deflected. This effect for spinning solids is now usually named after Gustav Magnus who investigated spinning cylinders. An analysis based on the Navier-Stokes equations for a rotating and translating sphere at low Reynolds numbers is due to Rubinov and Keller [2] who showed that the force is proportional to, and directed as the cross product of, the angular rotation rate $\boldsymbol{\omega}$ and translational velocity $\mathbf{v}$, $\boldsymbol{\omega} \times \mathbf{v}$. However, under some conditions the Magnus force is known to be in the opposite direction to that usually observed [3], [4]. This reverse Magnus effect is thus far only qualitatively understood.

Interest in the Magnus effect in free molecular flows, that is flows with high Knudsen number, has been sparked by a recent article by Borg, et al [5]. Using Maxwellian distribution functions, the paper presents a calculation of the transverse force on a spinning sphere translating in a rarefied gas. The direction of this force is shown to be opposite to that calculated in [2] for continuum flow. We will refer to this phenomenon as the inverse Magnus force. The calculation by Borg and his collaborators also includes the effect of heat transferred to the rotating sphere from the impinging particles. Experimental verification of the inverse Magnus force has not yet been reported. An intriguing possible application is to vortex dynamics in superfluids and superconductors, which is currently a subject of the debate in the solid state physics community (see, *e.g.* [6]).

A simpler and physically more transparent approach for understanding the high Knudsen number Magnus effect, presented here, is based on a particle dynamics model similar to, but crucially different from, that introduced by Newton (propositions 34 and 35, Book II of *Principia*) to calculate fluid resistance. Our alternative approach [7], is applicable not for a medium in which molecule-molecule interaction is the dominant effect, but to a rarefied gas where molecular interactions with boundaries are dominant. This model reproduces the result obtained by Borg et. al. [5] for the isothermal case (infinite conductivity of the rotating sphere). Further, it can be readily used to obtain both the total drag and lift forces on a spinning object of arbitrary shape translating through a rarefied gas.

Newton's model for fluid resistance "consists of equal particles freely disposed at equal distances from each other" impinging on a body such that their normal components of momenta are transferred to the body while their tangential components are preserved. This leads to the famous sine-squared law for the pressure coefficient over the surface of a body. (While this result is found not to be applicable to subsonic flow, it is fortuitously useful at hypersonic speeds.) Our model, different from Newton's, assumes: (i) mass of a gas particle is orders of magnitude smaller than the projectile mass, (ii) particle collisions with the body are perfectly elastic, and (iii) during collisions, gas particles acquire the fraction of the tangential velocity at the surface of the spinning body measured by α_τ, the Maxwellian accommodation coefficient.

We consider in turn a sphere, a cylinder, and right parallelepipeds of various sections, each in uniform translation and spinning about the primary axis of symmetry normal to the line of flight. Rather than dealing with the unsteady motion of a body moving through a cloud of particles, we take in each case a body fixed in space, but rotating about an axis of symmetry and exposed to an oncoming stream of uniformly dispersed molecules. For the sphere, cylinder, and right parallelepipeds of regular polygon sections with even number of faces, we show that the Magnus force is steady, opposite in direction to the usual force in continuum flow, and has the magnitude proportional to one-half the mass M of gas displaced by the body and the vector product of its angular and translational speeds:

$$\mathbf{F} = -\frac{1}{2} M \alpha_\tau \, \boldsymbol{\omega} \times \mathbf{v}.$$

For the sphere and the cylinder, the drag is steady and the lift (understood here as the sidewise force that does not disappear in the limit of vanishing accommodation coefficient, $\alpha_\tau \to 0$) is zero. As expected, the lift and drag are both unsteady for all parallelepipeds of regular polygon section. For parallelepipeds whose sections are polygons with odd-number of faces, the Magnus force is unsteady as well. Figure 1 shows the magnitude of the inverse Magnus force as a function of the rotation angle for the "worst case," namely that of a parallelepiped of triangular (equilateral) section.

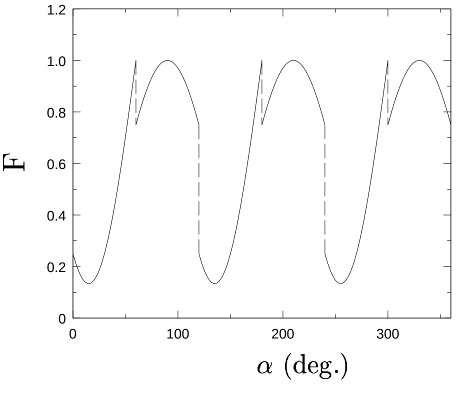

Figure 1: The normalized magnitude of the inverse Magnus force F as a function of rotation angle α for a parallelepiped of triangular, equilateral section. The force is unsteady, both wobbly and jerky.

Our results lead to the following general hypothesis. The inverse Magnus force for any planar object of convex section possessing two perpendicular axes of reflectional symmetry will be steady and given by the above expression. We consider also the case of spinning solids embedded in shear flow and discuss under what conditions the inverse Magnus force remains steady.

References

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