SELF-CONSISTENT METHODS IN THE PROBLEM OF ELASTIC WAVE PROPAGATION THROUGH MATRIX COMPOSITE MATRIALS

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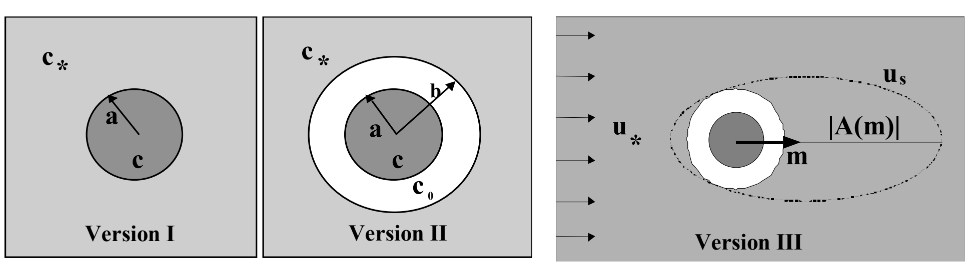
Summary The mathematical structures of two main self-consistent schemes used for the solution of the homogenization problem in the mechanics of heterogeneous media are analyzed and discussed. The dispersion equations for the wave numbers of the mean wave field that correspond to every of these schemes are developed for the composite medium with spherical particles and with unidirected cylindrical fibers. These equations serve for all frequencies of the monochromatic incident field and arbitrary properties and volume concentrations of inclusions. The results of the predictions of both methods are analyzed and compared.

THE MAIN HYPOTHETHIS OF SELF-CONSISTENT METHODS

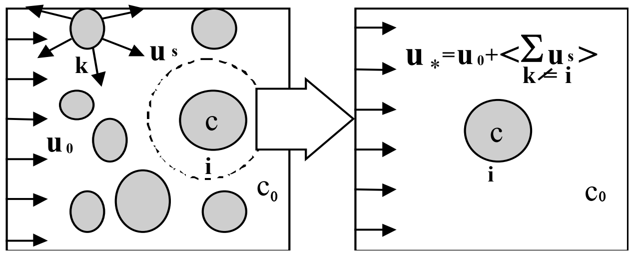
Self-consistent methods are widely used for the solution of the problem of monochromatic wave propagation through inhomogeneous media. In the case of the medium with random sets of isolated inclusions this problem cannot be solved exactly, and only approximate solutions are available. Among various approximations self-consistent methods have many important advantages. These methods are based on simple and physically clear hypotheses that reduce the problem of interactions between many inclusions in the composite to a diffraction problem for one inclusions. These hypothesis may be changed and corrected in order to improve the predictions of the methods. As a rule it is difficult to evaluate the area of application of these hypotheses, although in many cases self-consistent solutions are in a good agreement with experimental data. It is possible to point out two main self-consistent schemes: the effective field and effective medium methods. Let us consider the basic hypotheses of every of these methods.

Effective medium method (EMM). The effective medium method is in fact a group of methods based in a common hypothesis that for the evaluation of the wave field inside a typical inclusion the composite material outside some vicinity of this inclusion may be changed for a homogeneous medium with effective properties of all the composite. Let as consider a composite medium that consists of a matrix phase with the tensor of elastic properties C_0 and density ρ_0 and spherical inclusions with properties C and ρ. In the simplest version of the method (version I) for the calculation of the wave field inside an arbitrary inclusion it is assumed that this inclusion is embedded into a homogeneous medium with overall properties of all the composite C*. This hypothesis (H1) reduces the problem of interaction between many inclusions in the composite to the one particle problem. The second hypothesis (H2) is the assumption that the mean wave field in the composite coincides with the wave field in the effective medium (condition of self-consistency).
One can point out a more complex version of the method when the behavior of every inclusion is supposed to be coincided with the kernel of a layered inclusion embedded into the matrix with effective properties. The properties of the layer coincides with the properties of the matrix, and the size of the layer depends on the volume concentration p of inclusions $(a/b)^3 = p$ (version II). The condition of self-consistency in this case coincides with the hypothesis H2 of version I of the method.
Another version of the method (version III) is based on hypothesis H1 of version II but condition of self-consistency is formulated as follows. The properties of the effective medium is to be chosen in order to eliminate the forward amplitude **A(m)** of the field $\mathbf{u}_s$ scattered on the layered inclusion embedded into the effective medium, **m** is the wave normal of the mean wave field.



Analyses of this three versions of the EMM in the case of electromagnetic waves propagating through particulate composites is presented in [1] and for elastic waves propagation through fiber and particulate composites in [2,3,4].

Effetive field method (EFM). Another self-consistent method is based on the hypothesis that every inclusion on the composite behaves as an isolated one in the original matrix, and the presence of the surrounding inclusions is taken into account through the local external field (effective field) that acts on this inclusions. The main hypothesis of the EFM concern the structure of this field. The simplest ones assumes that the effective field is a plane wave that is the same for all the inclusions (quasicrystalline approximation). This hypothesis allows to write an integral equation for the effective field which kernel depends on a pair correlation function of the random field of inclusions. Thus, unlike the EMM, the predictions of the EFM depend on the spatial distribution of inclusions through the mentioned correlation function. The details of the application of this method are presented in [1] for electromagnetic waves and in [3] for elastic waves in fiber composites.



DISPERSION EQUATION FOR THE EFFECTIVE WAVE NUMBER

The hypothesis of the methods allow to develop dispersion equation for the effective wave vector **k*** of the mean wave field in the composite medium. This equation may be written in the following canonical form

$$\det[k_i^* C^*_{ijkl}(k^*)k_l^* - \rho_*(k^*)\omega^2\delta_{jk}] = 0.$$

For the EMM the effective dynamic elastic moduli tensor C* and effective dynamic density ρ^* have the following structure

$$C^*(k^*) = C_0 + p(C - C_0)H_C(k^*), \quad \rho_* = \rho_0 + p(\rho - \rho_1)H_\rho(k^*).$$

Here functions H_C and H_ρ are known from the solution of the one particle problem. For the EFM the functions H_C and H_ρ have more complex structure and depend on the following integrals

$$\int G(x)\Phi(x)e^{ik_*x}dx, \quad \int \nabla G(x)\Phi(x)e^{ik_*x}dx, \quad \int \nabla\nabla G(x)\Phi(x)e^{ik_*x}dx.$$

Here G(x) is the Green function of the homogeneous medium with the properties of the matrix, $\Phi(x)$ is a specific two point correlation function of the random field of inclusions.

LONG WAVE AND SHORT WAVE ASYMPTOTIC SOLUTIONS OF THE DISPERSION EQUATIONS

Asymptotic solutions of the dispersion equations of the EMM and EFM in the region of long (k*a<<1) and short (k*a>>1) may be obtained in analytical forms. In the long wave limit the dispersion equations give equations for the effective elastic properties of composite materials and the attenuation factors of the Rayleigh wave scattering (see[1-4]). In the short wave limit the methods predict that the phase velocity of the mean wave field coincides with the wave velocity in the matrix, and the attenuation factor γ does not depends on the properties of the inclusions and the matrix and γa=3p/4 for the EMM. For the EFM γa=f(p), where function f(p) depends on the pair correlation function Φ(x) and coincides with 3p/4 only for small volume concentrations of inclusions. The discussion of these properties of short wave asymptotics of the solutions of the dispersion equations see in[1-4].

CONCLUSION

The dispersion equations for the effective wave vector of the EMM and EFM are obtained and analyzed. Long and short wave solutions of these equations obtained in analytical forms. Numerical solutions of these equations are constructed in the wide region of frequencies of the incident field that covers the long middle and short regions. The main discrepancies and the sources of possible errors in the predictions of the methods are indicated, and the ways of corrections of these errors are discussed. It is shown that the EFM has some advantages in comparison with the EMM. The EFM allows to describe the influence of spatial distribution of inclusion on the mean wave field in composite, e.g., to evaluate the positions of photonic gaps in the frequency region for periodic composites. The latter result is impossible to obtain in the framework of the EMM.

References

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