A NEW NONLINEAR CONSTITUTIVE RELATION FOR MAGNETOSTRICTIVE MATERIAS

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SUMMARY A constitutive relation is suggested to describe the nonlinear and coupling features of magnetostrictive materials. It can effectively predict the magnetostrictive strains of the materials, the effect of the pre-stress on the maximum magnetostrictive strain and the magnetization, and the effect of the pre-stress and the magnetization on the Young's modulus. Its constitutive constants are material parameters and are easily measured in experiments.

INTRODUCTION

For magnetostrictive materials, many experiments have exhibited that their magneto-mechanical responses are nonlinear and coupled[1]. For example, for a Terfenol-D rod subjected to a compressive pre-stress, its the maximum value of the magnetostrictive strain is dependent on the pre-stress. Moreover, the Young's modulus of a Terfenol-D rod is not a constant, which is called as the ΔE effect. Many researchers have tried to establish the nonlinear constitutive relations for magnetostrictive materials, such as the SS Model[2], the HT Model and the DDS Model[3], and the D-H Model[4]. However, all of these models cannot exhibit good quantitative agreement with experimental data in the region of the high magnetic field, cannot simulate the experimental phenomenon that the maximum magnetostrictive strain of a Terfenol-D rod increases with the compressive pre-stresses, and cannot completely capture the main features of the ΔE effect in quality or in quantity. To overcome these deficiencies, a new nonlinear constitutive relation for magnetostrictive materials will be developed in this paper.

NEW CONSTITUTIVE RELATION

Several main steps are taken to derive the new constitutive relation given by Eq. (1). Firstly, the more high-order terms in the Taylor series expansion of Gibbs free energy function are reserved in the new relation than in the D-H Model so

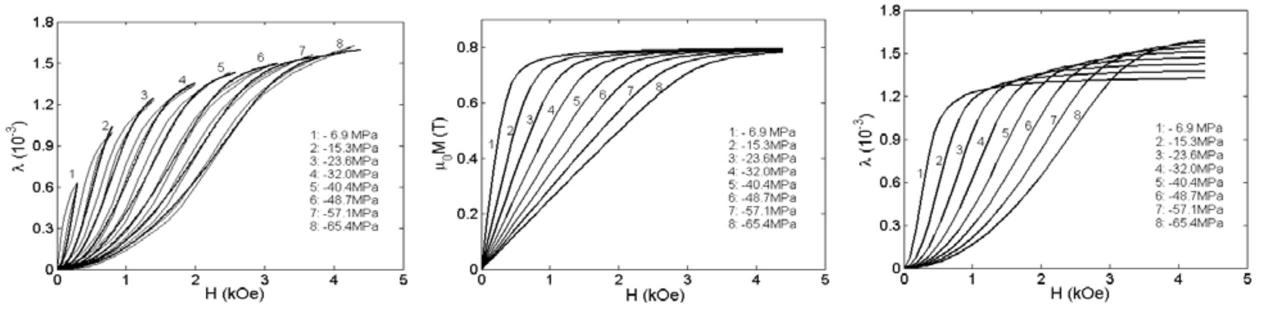
$$\varepsilon = \frac{\sigma}{E_s} + \begin{cases} \lambda_s \tanh(\dfrac{\sigma}{\sigma_s}) + [1 - \tanh(\dfrac{\sigma}{\sigma_s})]\dfrac{\lambda_s}{M_s^2}M^2 & \dfrac{\sigma}{\sigma_s} \geq 0 \\ \dfrac{\lambda_s}{2}\tanh(\dfrac{2\sigma}{\sigma_s}) + [1 - \dfrac{1}{2}\tanh(\dfrac{2\sigma}{\sigma_s})]\dfrac{\lambda_s}{M_s^2}M^2 & \dfrac{\sigma}{\sigma_s} < 0 \end{cases} \tag{1a}$$

$$H = \frac{1}{k} f^{-1}\left(\frac{M}{M_s}\right) - \begin{cases} 2\{\sigma - \sigma_s \ln[\cosh(\dfrac{\sigma}{\sigma_s})]\}\dfrac{\lambda_s}{\mu_0 M_s^2}M & \dfrac{\sigma}{\sigma_s} \geq 0 \\ 2\{\sigma - \dfrac{\sigma_s}{4}\ln[\cosh(\dfrac{2\sigma}{\sigma_s})]\}\dfrac{\lambda_s}{\mu_0 M_s^2}M & \dfrac{\sigma}{\sigma_s} < 0 \end{cases} \tag{1b}$$

that both the change of the Young's modulus with the stress in the case of $H = 0$ (or $M = 0$) and the effect of the pre-stresses on the maximum magnetostrictive strain may be described. Secondly, the nonlinear part of the elastic strain produced by the pre-stress is considered to be related to the rotation or movement of magnetic domains and can be approximated as a hyperbolic tangent function. Thirdly, according to magnetic domain analysis, the maximum magnetostrictive strain is expressed as the difference between the saturation magnetostrictive strain (or coefficient) and the nonlinear elastic strain produced by the pre-stress so that the new model can describe the change of the maximum magnetostrictive strain with the pre-stress.

QUANTITATIVE DISCUSSION AND CONCLUSIONS

The prediction values of magnetostrictive strain calculated by the new constitutive relation are plotted in Fig. 1. From Fig.1a, the calculated results are perfectly coincident with the Moffet et al's experimental data under the various compressive pre-stress levels not only in the region of the low and moderate magnetic fields but also in the region of the high field. From Fig. 1b, it can be found that the prediction values of the magnetization increase with the applied field under several pre-stress levels, and will reach a constant in the region of the high field. It means that the magnetization of the rod subjected to different pre-stresses will be saturated to the same level, i.e. the saturation magnetization. For a given magnetic field, the magnetization decreases with an increasing compressive pre-stress, which means that the axial compressive pre-stress makes the axial magnetization of the rod more difficult. When the applied magnetic field is in the high field region, or near the saturation field of the material, the predicted magnetostrictive strain of the rod, shown in Fig. 1c,will reach different saturation values for different pre-stresses. The larger the compressive pre-stress, the larger the maximum magnetostrictive strain. In the other words, the maximum magnetostrictive strain of the rod increases with the axial compressive pre-stress, which is obviously different from the characteristics in the region of the low and moderated fields. These predicted results are coincident with the observed experimental phenomena, and none of the previous constitutive relations (e.g. the SS Model, the HT Model, the DDS Model and the D-H Model) can capture all of them qualitatively or quantitatively. In addition, the predicted stress-strain relation is nonlinear and the calculated curves of Young's modulus versus compressive stress for some given magnetic bias fields are non-monotonic, which are very similar to those observed in experiments. Moreover, only one material parameter is added in the new relation by contrast with the D-H Model, and all the material parameters required by the new constitutive relation can easily be determined in experiments. So, the new constitutive relation suggested in this paper is convenient for engineering applications.



a:Comparison with the experimen tal data b: Prediction on Magnetization c: Prediction on Magnetostrictive strain

Fig. 1 Calculation values by the new constitutive equations (hysteresis loops: experimental;solid lines: theoretical)

Acknowledgements

This research was supported by a grant of the Key Fund of the National Natural Science Foundation of China (No. 10132010), and the Fund of Pre-Research for Key Basic Researches of the Ministry of Science and Technology of China. The authors gratefully acknowledge these supports.

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