# THERMAL BUCKLING OF ACTIVE COMPOSITE PLATES WITH SHAPE MEMORY ALLOY FIBERS

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<u>Summary</u> Micromechanically established constitutive equations for unidirectional composites with shape memory alloy fibers embedded in polymeric or metallic matrices are derived. These equations are subsequently employed to analyze the thermal buckling of rectangular composite plates. The shape memory alloy fibers are activated by a mechanical loading and unloading of the composite to an overall traction-free state, prior to the application of the thermal load. The present micro-macro-structural approach enables an accurate modelling that accounts for the interaction of shape memory alloy fibers with its surrounding rather than the commonly adopted simplified analyses.

### Introduction

Shape memory alloy (SMA) materials undergo phase transformation which is caused by the application of stress and/or temperature. At high temperatures, the material behavior is nonlinear and hysteretic but yet, yields at the end of a mechanical loading-unloading cycle the original stress-strain-free state (superelastic behavior). At lower temperatures, a mechanical loading-unloading results in a residual deformation which can be recovered by a temperature increase (shape memory effect). The latter effect can be utilized to control the behavior of structures in which SMA materials have been embedded.

Three-dimensional constitutive relations that can model the shape memory behavior have been presented by various investigators, see Boyd and Lagoudas[1], for example. Birman[2] reviewed various constitutive equations and discussed some applications of SMA materials.

One way of incorporating SMA materials in structures is by embedding SMA fibers in polymeric or metallic matrices to generate active composites. The analysis of the such composites should be based on a micromechanical approach in which the detailed interaction between the constituents is accounted for. In the presence of SMA constituents, the micromechanical analysis is necessarily nonlinear and path-dependent. In the presence of metallic materials additional nonlinear effects exist due to their inelastic behavior.

The analysis of composite structures that involve embedded SMA materials should be based on constitutive relations that have been established by a micromechanical approach. Alternatively, several investigators (Tawfik et al.[3], for example) adopted a simplified approach according to which the detailed interaction of the SMA material with its surrounding is neglected and its effect, as deduced from the behavior of the monolithic SMA material, is incorporated.

## Analysis

In the present paper, the thermal buckling of plates consisting of active composites is considered. The structural analysis is based on a micromechanical model referred to as the Generalized Method of Cells (GMC) (Aboudi[4]) which establishes constitutive relations of the inelastic composite. Due to the complicated state of stress, strain and temperature resulting from the structural behavior, these constitutive relations are used at every point of the plate. Consequently, such an analysis forms a micro-macro-structural approach.

The modelling of Boyd and Lagoudas[1] is employed to predict the behavior of the SMA phase. This three-dimensional model is capable of producing the superelastic and the shape memory effects. Two kinds of matrices in which the SMA fibers can be embedded are considered in the present paper: (1) a polymeric matrix (such as epoxy) which is assumed to behave as a linearly elastic material and, (2) a metallic matrix (such as aluminum alloy) which is assumed to behave as an elastic-viscoplastic work-hardening material (Bodner [5]).

In order to utilize the shape memory effect, the composite is activated by an isothermal mechanical loading followed by unloading to a state of overall zero stresses associated with non-zero overall strains. Due to the presence of SMA fibers, a subsequent applied thermal loading of traction-free composite plate reduces the overall residual strains. On the other hand, in a pre-loaded composite plate in which the in-plane displacements of the edges are restrained, the heat induced reduction of the residual strains is prevented and recovery tensile

stresses can develop. These tensile stresses, caused by the phase transformation, reduce or even overcome the compressive stresses generated by thermal expansion. Thus the ability to improve the buckling behavior of a structure can be achieved by utilizing the shape memory effect. It is obvious that an activating pre-loading cycle when applied to the active composite, generates internal residual stress and strain fields in the composite constituents. The micromechanical approach enables the incorporation of the effect of these residual micro fields on the structural behavior under the subsequent thermal loading.

## Application

Results are presented that exhibit the buckling temperature of SMA/Epoxy and SMA/Aluminum composites, as well as that of the monolithic SMA, the epoxy and the inelastic aluminum constituents. In particular, the effect of the three-dimensional state of stress that exists in the SMA and matrix phases on the overall composite behavior is shown. This effect can, for example, be deduced from Fig. 1 in which the critical temperature increase,  $\Delta T$ , versus the amplitude of the activating pre-loading,  $\epsilon^0$ , is displayed for various square simply supported plates.

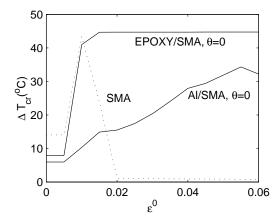


Fig 1: Variation of the buckling temperature with the pre-strain.

#### References

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