XXI ICTAM, 15–21 August 2004, Warsaw, Poland

INFLUENCE OF REMAINING CHAOS ON CONVERGENCE OF SOLUTIONS IN TIME DELAYED FEEDBACK CONTROLLED DUFFING SYSTEM

Kohei YAMASUE*, Takashi HIKIHARA*
*Department of Electrical Engineering, Kyoto University, Katsura, Nishikyo, Kyoto 615-8510, Japan

Summary Time delayed feedback control is well-known as a practical method for stabilizing unstable periodic orbits embedded in chaotic attractors. However, its control performance has not been clarified with relation to global dynamics of controlled systems. In this paper, we numerically discuss the influence of a remaining chaotic invariant set under control for chaos of Two-well Duffing system. We reveal that it causes highly complicated domain of attraction for target orbits and long chaotic transient before the convergence.

Since the pioneer work by Ott, Grebogi and Yorke [1], methods for stabilizing unstable periodic orbits embedded in chaotic attractors have been extensively studied in the field of nonlinear dynamics [2]. Among several advanced methods, the time delayed feedback control method has an ability to stabilize target unstable periodic orbits by using the difference between present output signals and past ones of controlled objects [3]. The control method is easily implemented without the exact model of the controlled object nor complicated computer processing for reconstruction of the underlying dynamics. For these practical advantages, the control method has been successfully applied to experimental chaotic systems in various fields of research including electronic circuits [4], laser systems [5], mechanical oscillators [6] and chemical systems [7] and so on. In addition, its theory has developed based on local stability analysis around the target unstable periodic orbits. One of the important results is the derivation of odd number condition. The odd number condition gives a class of unstable periodic orbits which can not be stabilized by the control method [8, 9, 10]. Against this negative condition, Pyragas has recently improved the control method [11].

On the other hand, there still remain open problems for clarification of control performance [13]. In particular, the following two problems are here focused on with relation to the convergence of states to target orbits. One is the domain of attraction for target orbits under control [13]. Its tract and structure are closely related to practical problems such as deciding onset timing of control and estimating effects of noise. It also gives information necessary for targeting scheme when multiple orbits coexist in the controlled system. Another is transient behavior after onset time of control. It is expected that the controlled system quickly converges toward the target orbit without irregular behavior before convergence. While these problems are quite important for engineering use of the control method, there was no detailed discussion about them. The problems are directly connected to global dynamics of the controlled system with time delay. Therefore it is essential to consider the structure of the phase space in function space, which has infinite dimension.

In this paper, we numerically discuss the global structure of the time delayed feedback controlled Two-well Duffing system. The discussion is based on the existence of the one dimensional global unstable manifold originating from a directly unstable fixed point, which corresponds to an unstable periodic orbit which can not be stabilized due to the odd number condition. The homoclinic intersection of its unstable manifold and stable manifold generates a chaotic invariant set in the uncontrolled system [14, 15]. In the following, the presence of the chaotic invariant set in the controlled system and its influence on the domain of attraction and transient behavior are discussed.

The Two-well Duffing system is a model of the first-mode vibration in magnet-elastic beam system with sinusoidal forcing [14]. This paper deals with a Two-well Duffing system controlled with use of velocity component for delayed feedback. The system is described by the following delay differential equation : $\ddot{x}(t) + \delta\dot{x}(t) + x(t)^3 - x(t) = A\cos\omega t + u(t)$, where $x(t)$ and $\dot{x}(t)$ denote the displacement and velocity of the Two-well Duffing system, respectively. The $u(t)$ indicates control signal simply given as $u(t) = K[\dot{x}(t-\tau) - \dot{x}(t)]\,;\, t \geq t_0$, where K is feedback gain and t_0 onset time of control. The τ is time delay, which is adjusted to the period of target unstable periodic orbits embedded in chaotic attractors. The $u(t)$ obviously keeps null before onset time of control. Once control turns on, the control signal converges to null according to the stabilization of the system to one of the target orbits. As a result of the convergence, the controlled system degenerates from the infinite dimensional system with time delay to the original two dimensional system. Since the domain of attraction and transient behavior are characterized by the infinite dimensional system under remarkable control input, their treatment is obviously beyond the scope of local stability analysis of target orbits. In the following, the parameter is fixed at $(\delta,\, A,\, \omega) = (0.16,\, 0.27, 1.0)$, where the uncontrolled system generates the chaotic attractor [15]. δ here denotes damping coefficient. A represents forcing amplitude and ω the frequency. The dynamics under $\omega = 1.0$ was summarized in [15]. τ is adjusted to 2π for stabilizing two symmetric inversely unstable periodic orbits with period-2π. Note that the number and location of orbits with period-2π keeps under control [16]. The controlled system has three unstable periodic orbits with period-2π [15], as shown in Fig. 1. Two of them are the target orbits (denoted by $^1\mathrm{I}$ and $^1\mathrm{I}'$) mentioned above. They are easily stabilized by the control. The other is a directly unstable periodic orbit (denoted by $^1\mathrm{D}$) which can not be stabilized due to the odd number condition. The $^1\mathrm{D}$ has one dimensional global unstable manifold in the controlled system, because it possesses only one real characteristic multiplier greater than unity. In the following, the global structure of the controlled system is discussed with relation to this unstable manifold.

Figure 2 shows the unstable manifold of $^1\mathrm{D}$ in the controlled system. When $K = 0$, the unstable manifold is obviously identical to that in the uncontrolled system, as shown in Fig. 2(a). The closure of the unstable manifold coincides with the only one chaotic attractor shown by stroboscopic points. Since the unstable manifold of $^1\mathrm{D}$ transversely intersects the stable manifold of $^1\mathrm{D}$ [14, 15], the controlled system has a chaotic invariant set. Then the regions of phase space are stretched and folded along the unstable manifold with evolution of time. Figure 2(b) shows the unstable manifold at $K = 0.75$. At this feedback gain, the two target orbits are stable. They are denoted by $^1\mathsf{S}$ and $^1\mathsf{S}'$ in Fig. 2(b). The unstable manifold appears to intersect itself, because the unstable manifold on stroboscopic map is the projection from the function space. The two branches of the unstable manifold start from $^1\mathsf{D}$ in the opposite direction from each other. However they are folded and then come close to $^1\mathsf{D}$ again parallel to the branches itself. The branches further grows in the opposite direction from each other. The regions of phase space is stretched near $^1\mathsf{D}$ and then folded around left and right hand sides in Fig. 2(b) with evolution of time. It implies that the phase space at $K = 0.75$ inherits the characteristic of the chaotic dynamics obtained at $K = 0$, although there is a difference that the phase space is here defined in function space under control signal. With the same approach, we also confirmed that further increase of feedback gain brings some global bifurcations which completely break the homoclinic intersection obtained in the uncontrolled system. After this bifurcation, the unstable manifold becomes quite simple compared with that in Fig. 2(b). Therefore, the unstable manifold in Fig. 2(b) shows the remain of chaotic invariant set in the controlled system, while it is not an attractor because of the presence of stabilized target orbits. The trajectory after onset time of control wanders between $^1\mathsf{S}$ and $^1\mathsf{S}'$ along the unstable manifold, as shown by the stroboscopic points in Fig. 2(b). Its change in time is shown in a window of Fig. 2(b). It shows that the controlled trajectory irregularly goes back and force between $^1\mathsf{S}$ and $^1\mathsf{S}'$ many times, before final convergence to $^1\mathsf{S}$. Then, the steady state obtained by the control is not predictable because of this long and irregular transient behavior. Figure 3 shows the classification of stroboscopic points with steady states. Each point here corresponds to different onset timings of control which is taken every period-2π. Before each onset time, the system generates the chaotic attractor, which coincides with the closure of the unstable manifold at $K = 0$ (See Fig. 2(a)). The points are classified with green and red color, which imply convergence to $^1\mathsf{S}$ and $^1\mathsf{S}'$, respectively. The green and red points are finely mixed one another all over the chaotic attractor. It implies that the steady states almost randomly alternate between $^1\mathsf{S}$ and $^1\mathsf{S}'$ with onset timing of control and influence of external disturbance. In addition, since the two types of points are mixed even in the neighborhood of the target orbits, the controlled system possibly converges to the target orbit in the opposite side from the other target, near which control started. Therefore, it is inevitable that the targeting scheme based on linearisation has no possibility of effective convergence. These characteristics are obviously disadvantage for engineering use of the control method. However, no detail discussion for the characteristics has been obtained.

In summary, the original chaotic invariant set possibly remains in the controlled Two-well Duffing system, even when the feedback gain is enough large to stabilize the target orbits. The remain of the chaotic invariant set causes highly complicated domain of attraction of two symmetric target orbits and long chaotic transient before convergence to them. It shows that the further extension of the control method including targeting scheme and design of feedback gain should be established in the light of the global structure in function space.

References

[1] Ott E., Grebogi C. and Yorke J.A.: Controlling chaos. *Phys. Rev. Lett.* **64**:1196–1199, 1990.
[2] Schuster H.G., ed.:Handbook of Chaos Control. Wiley-VCh, 1999.
[3] Pyragas K.: Continuous control of chaos by self-controlling feedback. *Phys. Lett. A* **170**:421–428, 1992.
[4] Pyragas K. and Tamaševičius: Experimental control of chaos by delayed self-controlling feedback. *Phys. Lett. A* **180**:99–102, 1993.
[5] Bielawski S., Derozier D. and Glorieux P.: Controlling unstable periodic orbits by a delayed continuous feedback.*Phys. Rev. E* **49**:R971–R974, 1994.
[6] Hikihara T. and Kawagoshi T.: An experimental study on stabilization of unstable periodic motion in magnet-elastic chaos. *Phys. Lett. A* **211**:29–36, 1996.
[7] Parmananda P., Madrigal R., Rivera M., Nyikos L., Kiss I. Z. and Gáspár V.: Stabilization of unstable steady states and periodic orbits in an electrochemical system using delayed-feedback control.*Phys. Rev. E* **59**:5266–5271, 1999.
[8] Ushio T.: Limitation of delayed feedback control in non-linear discrete-time dynamical systems. *IEEE Trans. CAS-I* **43**:815–816, 1996.
[9] Just W., Bernard T., Ostheimer M., Reibold E. and Benner H.: Mechanism of time-delayed feedback control. *Phys. Rev. Lett.* **78**:203–206, 1997.
[10] Nakajima H. and Ueda Y.: Limitation of generalized delayed feedback control. *Physica D* **111**:143–150, 1998.
[11] Pyragas K.: Control of chaos via an unstable delayed feedback controller. *Phys. Rev. Lett.* **86**:2265–2268, 2001.
[12] Hikihara T. and Ueda Y.: An expansion of system with time delayed feedback control into spatio-temporal state space. *Chaos* **9**:887–892, 1999.
[13] Just W., Reibold E., Kacperski K., Fronczak P., Holyst J.A. and Benner H.: Influence of stable Floquet exponents on time-delayed feedback control. *Phys. Rev. E* **61**:5045–5056, 2000.
[14] Moon F.C. and Holmes P.J.: A magnetoelastic strange attractor. *J. Sound Vib.* **65**:275–296, 1979.
[15] Ueda Y., Nakajima H., Hikihara T. and Stewart H.B.:Forced two-well potential Duffing's oscillator, in Dynamical Systems Approaches to Non-linear Problems in Systems and Circuits (Eds. by Tzafestas S.G. and Borne P.):173–180, SIAM, 1988.
[16] Nakajima H.: On analytical property of delayed feedback control of chaos. *Phys. Lett. A* **232**:207–210, 1997.

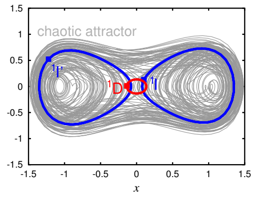
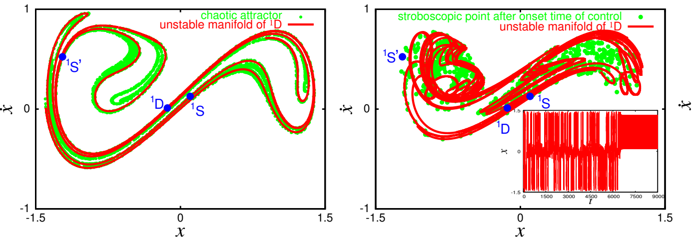


Figure 1. Target unstable periodic orbits (denoted by $^1\mathsf{I}$ and $^1\mathsf{I}'$) embedded in chaotic attractor



(a) $K = 0$

(b) $K = 0.75$

Figure 2. Unstable manifold of $^1\mathsf{D}$ projected on stroboscopic map

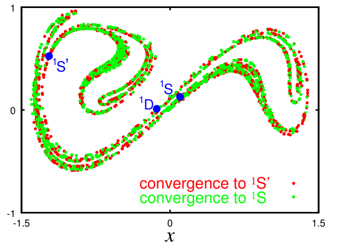


Figure 3. Classification of stroboscopic points with stabilized states.