XXI ICTAM, 15–21 August 2004, Warsaw, Poland

THE TRAILING EDGE PROBLEM FOR MIXED CONVECTION FLOW PAST A HORIZONTAL PLATE

Herbert Steinrück, Ljubomir Savic
Institute of Fluid Dynamics and Heat Transfer, Vienna University of Technology, Resselgasse 3, A-1040 Vienna, Austria

Summary
The influence of buoyancy onto the boundary-layer flow past a horizontal plate aligned parallel to a uniform free stream is characterized by the buoyancy parameter $K = \mathrm{Gr}/\mathrm{Re}^{5/2}$ where Gr and Re are the Grashof number and the Reynolds number, respectively.
An asymptotic analysis of the complete flow field including potential flow, boundary layer, wake and interaction region near the trailing edge will be given for small buoyancy parameters and large Reynolds numbers in the distinguished limit $K\mathrm{Re}^{1/4} = \mathrm{O}(1)$.

INTRODUCTION

Laminar two-dimensional mixed convection flow along a horizontal heated plate (plate temperature θ_w) aligned parallel to a uniform free stream (velocity u_∞, temperature θ_∞) will be investigated for large Reynolds Re and large Grashof numbers Gr in a distinguished limit. In previous papers [1], [3] the boundary layer flow for $K = \mathrm{Gr}/\mathrm{Re}^{5/2} = O(1)$ has been investigated. Since the buoyancy force is orthogonal to the main flow direction the boundary layer flow is affected only indirectly by the buoyancy force. The buoyancy force is compensated by a hydro-static pressure distribution. Since the boundary-layer thickness is not uniform this hydro-static pressure distribution is not uniform either resulting in a horizontal pressure gradient accelerating or decelerating the boundary layer flow. Thus we speak of a favorable or an adverse buoyancy. We note that for a fluid with a positive isobaric expansion coefficient buoyancy affects the boundary layer flow on the upper side favorably and on the lower side adversely for a heated plate.
In case of adverse buoyancy the boundary-layer equations have the remarkable properties:
(i) They have a one parametric family of solutions.
(ii) Perturbations can propagate upstream in the boundary layer flow without interaction with the outer potential flow.
Both properties indicate that the boundary-layer flow has to be analyzed in context with the global flow field. A special role will play the trailing edge region where we will apply triple deck methods. Thus the buoyancy parameter K has to be chosen appropriately: $K = \kappa\mathrm{Re}^{-1/4}$ with the reduced buoyancy parameter $\kappa = O(1)$.

ASYMPTOTIC ANALYSIS

For the analysis the coordinates x and y parallel and normal to the plate, respectively, are made dimensionless with the plate length. Velocities are referred to the velocity of the free stream and temperature difference are referred to $\theta_w - \theta_\infty$. According to triple deck theory. We have to decompose the flow field into the different regions using the scalings listed in table 1 and sketched in figure 1.

outer flow field (potential flow)	x	y
boundary layer	x	$\bar{y} = y\mathrm{Re}^{1/2}$
trailing edge upper deck	$X = x\mathrm{Re}^{3/8}$	$Y = y\mathrm{Re}^{3/8}$
trailing edge main deck	$X = x\mathrm{Re}^{3/8}$	$\bar{y} = y\mathrm{Re}^{4/8}$
trailing edge lower deck	$X = x\mathrm{Re}^{3/8}$	$\hat{y} = y\mathrm{Re}^{5/8}$
wake	x	$\bar{y} = y\mathrm{Re}^{1/2}$

Table 1: Scalings of the different flow regions

Boundary-layer

The expansion of the flow and temperature field in the boundary-layer is given by

$$u(x,\eta) \sim \bar{u}_0(\eta) + K\bar{u}_1(x,\bar{y}) + ..., \quad \theta(x,\eta) \sim \bar{\theta}_0(\eta) + K\bar{\theta}_1(x,\bar{y}) + ... \quad \text{with} \quad \eta = \bar{y}/\sqrt{x+1}, \tag{1}$$

where $\bar{u}_0$ is the Blasius velocity profile. Note that the leading order terms $\bar{u}_0$, $\bar{\theta}_0$ are independent of buoyancy. The first order terms $\bar{u}_1$ and $\bar{\theta}_1$ are affected by buoyancy in two ways: Firstly by the hydro-static pressure distribution in the boundary layer and secondly by the first order correction of the potential flow.

Wake

In the wake the solution has a similar expansion as in the boundary layer. We note that across the wake there is a hydro-static pressures difference :

$$\Delta p_h(x) = K(p_1(x,0+) - p_1(x,0-)) = K\int_{-\infty}^{\infty} \bar{\theta}_0(x,\bar{y})\,\mathrm{d}\bar{y} \tag{2}$$

Moreover a curvature of the centerline of the wake has to be considered.

Potential flow

In contrast to usual boundary-layer fbw problems the first order correction of the potential fbw is not induced by the displacement of the boundary but by the hydrostatic pressure difference across the wake which is of order $K = \kappa \mathrm{Re}^{-1/4}$. Thus the stream function ψ of the outer fbw field has the expansion $\psi(x, y) = y + K\psi_1(x, y) + ...$ where ψ_1 satisfies the boundary conditions:

$$\psi_{1,x}(x, 0) = 0 \quad \text{for} \quad -1 < x < 0, \qquad \psi_{1,y}(x, 0) = -\int_{-\infty}^{\infty} \bar{\theta}_0(x, \bar{y})\, \mathrm{d}\bar{y} \quad \text{for} \quad 0 < x. \tag{3}$$

In [2] the potential fbw problem for $\psi_y(x, 0) = const$ for $x > 0$ has been solved. It turned out that this potential fbw problem for the first order correction has no solution if considered in the entire plane. The singularity of the potential fbw can be removed if one considers the fbw between two distant walls [2].

Trailing Edge

Near the trailing edge the usual triple deck scaling applies [5]. We expand all fbw quantities with respect to Re and K with $\mathrm{KRe}^{1/4} = \mathrm{O}(1)$. Thus to leading order we recover the well known trailing edge problem for a plate in a parallel uniform free stream [4]. Buoyancy influences only the second order terms, which can be decomposed into terms that are affected by buoyancy and terms which are independent of K:

$$u = 1 + \mathrm{Re}^{-1/4} u_s + \mathrm{Re}^{-3/8}(u_2 + K\mathrm{Re}^{1/4}\Delta u_2) + ... \tag{4}$$

Note that subscript s denotes the solution of the well-known trailing edge problem [4]. For the buoyancy correction terms we obtain the linearized triple deck problem:

$$u_s \Delta u_{2,X} + \Delta u_2 u_{s,X} + v_s \Delta u_{2,Y} + u_{2,Y} \Delta v_2 = -\Delta p_{2,X} + \Delta u_{2,YY}, \quad \Delta u_{2,X} + \Delta v_{2,Y} = 0 \tag{5}$$

with the interaction law

$$\Delta p_2 = A_s(X) - (1 + h(-X)) a_s |X|^{1/3} + \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\Delta A_2'(\xi) + \sqrt{3} a_s \xi^{1/3} h(\xi)}{X - \xi}\, \mathrm{d}\xi. \tag{6}$$

Here $X = x\mathrm{Re}^{3/8}$ and $Y = y\mathrm{Re}^{5/8}$ denote the lower deck variables, respectively. The leading order term of the displacement thickness A_s has the expansion $A_s \sim a_s X^{1/3}$ for $X \to \infty$. At the plate $X < 0, Y = 0$ the no slip boundary conditions hold and since Δu_2 and Δp_2 are antisymmetric with respect to $Y = 0$ we obtain the boundary conditions $\Delta u_2(X, 0) = 0, \Delta p_2(X, 0) = 0$ in the wake ($X > 0$). The matching condition to the main deck is $\Delta u_2(X, \infty) = \Delta A_2(X)$ where $\Delta A_2(X)$ is the correction of the displacement thickness due to buoyancy.

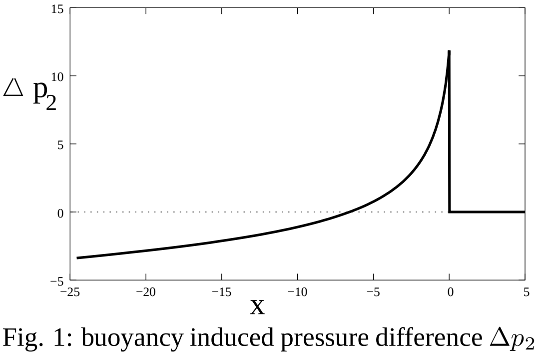

Fig. 1: buoyancy induced pressure difference Δp_2

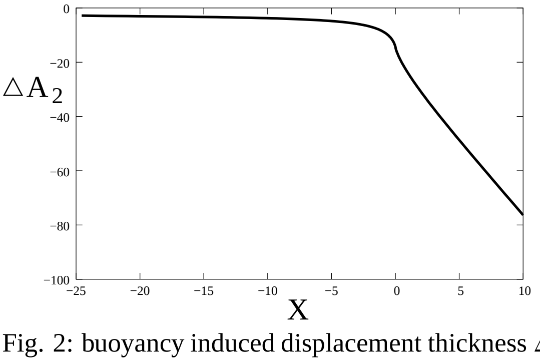

Fig. 2: buoyancy induced displacement thickness ΔA_2

Using Veldman's method [5] the triple deck problem and the buoyancy correction problem (4)-(6) is solved numerically. In figures 1 and 2 the pressure correction and the displacement correction are shown, respectively. The pressure has a discontinuity at the trailing edge. This is reflected in the vertical tangent of the displacement thickness ΔA_2 at $X = 0$. Thus there is a fbw around the trailing edge.

References

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