A NEW ENERGY-BASED ELASTOPLASTIC DAMAGE MODEL FOR CONCRETE

Jianying Wu*, Jie Li *
*Building Engineering Department, Tongji University, Shanghai, China

Summary In this paper, a new elastoplastic damage model is proposed, in which tensile damage variable and shear damage variable are adopted to describe the degradation of macro-mechanical properties of concrete. Within the framework of continuum damage mechanics, an elegant constitutive law which is the same as the effective stress concept is obtained. The plastic Helmholtz free energy is accounted for the damage growth, and the damage criteria are based on the elastoplastic damage energy release rates, which is consistent with thermodynamics theory. The evolutions of damage variables and plastic strains are established based on the normal rule and the effective stress space plasticity, respectively. Some pertinent computational aspects concerning the numerical algorithm are discussed. The model has been coded into a general finite element program which is capable of predicting the nonlinear behaviours of concrete under different stress states, whose predictive results demonstrate its adequate accuracy for the intended applications.

INTRODUCTION

Damage criteria are of significant importance to a damage model. In the current literatures, several different methods, including the equivalent strain-based, certain stress-based, and damage energy release rate-based, are adopted to define the damage criteria. Just as in classical plasticity most of yield criteria are functions of the invariants of stress tensor which is conjugated to plastic strain tensor, being the thermodynamic forces conjugated to damage variables, the damage energy release rate based damage criteria are more thermodynamically consistent[1],[2].However, the energy release rates based damage model of Mazars[1] and Ju[2] can predict neither the strength enhance in compression-compression stress states, nor the softening effect in the tension-compression stress states.

This paper attempts to present a novel rate-independent elastoplastic damage model for concrete, which not only is consistent with thermodynamics theory, but also fits the experimental results fairly well.

BASIC FORMULATIONS

Damage mechanisms

From the experimental phenomena observed, three types of mechanisms leading to the failure of concrete can be distinguished: tensile damage, shear damage, and compressive consolidation which results in the collapse of the micro-porous structure of the cement matrix under high hydrostatic compression and is not considered in present model.

Thus as generally accepted, there are two basic but distinct damage mechanisms that result in the degradation of macro-mechanical properties of concrete, which can be described by tensile damage variable d^+ and shear damage variable d^-, corresponding to pure tension and pure compression, respectively.

Elastoplastic Damage model

The classical effective stress is generally defined and can be decomposed as following[2],[3]:

$$\bar{\boldsymbol{\sigma}} = \mathbf{C}_0 : \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p\right) = \bar{\boldsymbol{\sigma}}^+ + \bar{\boldsymbol{\sigma}}^- ; \quad \bar{\boldsymbol{\sigma}}^+ = \mathbf{P}^+ : \bar{\boldsymbol{\sigma}} ; \quad \bar{\boldsymbol{\sigma}}^- = \bar{\boldsymbol{\sigma}} - \bar{\boldsymbol{\sigma}}^+ = \mathbf{P}^- : \bar{\boldsymbol{\sigma}} \tag{1}$$

and the ratio of effective stress can also be similarly decomposed as:

$$\dot{\bar{\boldsymbol{\sigma}}} = \mathbf{C}_0 : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p\right) = \dot{\bar{\boldsymbol{\sigma}}}^+ + \dot{\bar{\boldsymbol{\sigma}}}^- ; \quad \dot{\bar{\boldsymbol{\sigma}}}^+ = \mathbf{Q}^+ : \dot{\bar{\boldsymbol{\sigma}}} ; \quad \dot{\bar{\boldsymbol{\sigma}}}^- = \dot{\bar{\boldsymbol{\sigma}}} - \dot{\bar{\boldsymbol{\sigma}}}^+ = \mathbf{Q}^- : \dot{\bar{\boldsymbol{\sigma}}} \tag{2}$$

where $\mathbf{C_0}$ denotes the usual fourth-order isotropic linear-elastic constitutive tensor; $\mathbf{P}^+$ and $\mathbf{P}^-$, $\mathbf{Q}^+$ and $\mathbf{Q}^-$ are all fourth-order symmetric tensor, introduced as the positive and negative projection tensors of $\bar{\boldsymbol{\sigma}}$ and $\dot{\bar{\boldsymbol{\sigma}}}$, respectively.

With elastic Helmholtz free energy defined as the degradation of initial free energy[2], relations can be derived as:

$$\boldsymbol{\sigma} = \left(\mathbf{I} - \mathbf{D}\right) : \bar{\boldsymbol{\sigma}} ; \quad \mathbf{D} = d^+ \mathbf{P}^+ + d^- \mathbf{P}^- \tag{3}$$

which is the same as the classical effective stress concept, and $\mathbf{D}$ is the fourth-order damage tensor.

Damage criteria

After the plastic free energy is considered, the following damage energy release rates can be obtained:

$$Y^+ = \sqrt{E_0 \left(\bar{\boldsymbol{\sigma}}^+ : \boldsymbol{\Lambda}_0 : \bar{\boldsymbol{\sigma}}\right)} ; \quad Y^- = \alpha \bar{I}_1 + \sqrt{3 \bar{J}_2} \tag{4}$$

Then the damage criteria can be established as[2],[3]:

$$g^{\pm}(Y_n^{\pm}, r_n^{\pm}) = Y_n^{\pm} - r_n^{\pm} \leq 0 \tag{5}$$

In eqn. (5), the thresholds of damage energy release rates $r_n^{\pm}$ are determined as:

$$r_n^{\pm} = \max\left\{ r_0^{\pm}, \max_{\tau\in[0,n]} Y_\tau^{\pm} \right\} \tag{6}$$

where $r_0^{\pm}$ are the initial thresholds which can be expressed applying eqn.(4) under axial tension and axial compression.

Evolution laws of damage variables

Applying the normal rule, analogous to the plasticity, following evolution laws of damage variables are established:

$$\dot{d}^{\pm} = \dot{\lambda}^{d^{\pm}} h^{d^{\pm}}; \dot{\lambda}^{d^{\pm}} = \dot{r}_n^{\pm} = \dot{Y}_n^{\pm}; g^{\pm}\left(Y_n^{\pm}, r_n^{\pm}\right) \le 0, \dot{\lambda}^{d^{\pm}} \ge 0, \dot{\lambda}^{d^{\pm}} g^{\pm}\left(Y_n^{\pm}, r_n^{\pm}\right) \le 0 \tag{7}$$

Once proper expressions for the two damage variables are given, the above laws can be determined easily. In present model, for d^{+} and d^{-}, the expressions of Oliver (1990) and Faria et al. [3] are adopted.

Plastic strains considerations

The plastic strains can be obtained based on the effective stress space plasticity:

$$\dot{\boldsymbol{\varepsilon}}^{p} = \dot{\lambda}^{p} \partial_{\bar{\boldsymbol{\sigma}}} G^{p}; \dot{\boldsymbol{\kappa}} = \dot{\lambda}^{p} \mathbf{h}^{p}; F^{p}(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) \le 0, \dot{\lambda}^{p} \ge 0, \dot{\lambda}^{p} F^{p}(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) \le 0 \tag{8}$$

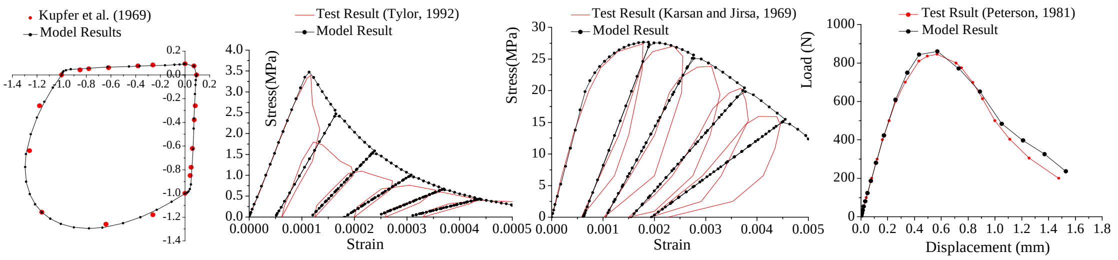
In present model, the plastic yield function of Lee and Fenvas[4] for F^{p} and the usual Drucker-Prager type function for G^{p} are adopted. The hardening parameters used here are referred to the equivalent plastic stains under tension and compression, respectively.

NUMERICAL ALGORITHM

In accordance with the operator-split method, the plastic flow and damage evolutions are decoupled, and three steps numerical system, i.e., the elastic-trial part, the plastic-corrector part and the damage-corrector part, are established. In the elastic-trial and the plastic corrector steps, for the yield function adopted, a new and uniform spectral-decomposition format of the classical return-mapping algorithm is developed and improved upon Lee and Fenvas[4], in which the principal vectors of the trial effective stress tensor remain unchanged in the plastic-corrector step, and the principal values can be easily determined based on the simple relation between the updated and the trial ones.

EXPERIMENTAL VERIFICATIONS

The present model and corresponding algorithm are coded into a finite element program, which is used to simulate the experimental results of concrete material and structures tests. Results presented following are the biaxial tests of Kupfer et al., cyclic tensile test of Tylor, cyclic compressive test of Karson and Jirsa, and the notched beam test of Peterson.



CONCLUSIONS

In this paper, to describe the degradation of macro-mechanical properties of concrete, tensile and shear damage variables are adopted, whose damage criteria are established based on the elastoplastic damage energy release rates upon considering the plastic free energy. The plastic strains are determined in accordance with the effective stress space plasticity. Thus an elegant elastoplastic damage model is derived, whose numerical algorithm is developed in the spectral-decomposition format of the classical return-mapping algorithm by decoupling the plastic flow and damage evolutions. Finally the model and its algorithm are applied to simulate several typical experimental tests, and the good agreements between predictive results and the test data verify their validity. Applying the present model to reinforced concrete structures will be forthcoming.

References

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