SELF SIMILAR DYNAMIC EXPANSION OF A SPHERICAL CAVITY IN ELASTOPLASTIC MEDIA

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Extended Summary

The dynamic Elastoplastic field induced by a pressurized spherical cavity expanding in an infinite medium is widely used in simulating penetration phenomena. The simplicity offered by the spherical symmetric pattern of the deforming material leads to fairly simple, yet accurate, expressions for key parameters like the resisting force and penetration depth. An extensive review of earlier work has been given by Hopkins [1] with emphasis on incompressible Mises elastic/perfectly-plastic models. Elastic compressibility in dynamic spherical cavity expansion has been considered by Forrestal and Luk [2] for an elastic/perfectly-plastic Tresca solid along with linear pressure-dilatation relation.

In the study lecture we attempt to present a unified treatment of dynamic spherical expansion in a pressure sensitive elastoplastic medium. Material response is modeled by the hypoelastic theory for the Drucker-Prager solid [3] with a non-associated flow rule. The theory accounts for both elastic and plastic compressibility and allows for arbitrary strain-hardening in the plastic range.

The lecture begins with a brief exposition of the governing dynamic field equations following the quasi-static spherical cavity expansion analysis in [3]. Assuming a self-similar expansion field we show that the governing system consists of four ordinary differential equations with two stress components, radial velocity and density as unknowns.

Next we examine the external elastic field, which is expected to develop at a distance from the cavity prior to plastic yielding. The governing system is reduced here to just two equations for the stresses, which can be further simplified, under the assumption of small elastic stresses, to the standard linear elastic model. A new observation that emerges from the elastic solution is the possible existence of a compressive elastic zone where yielding is prevented since the effective stress remains negative. A simple expression for the location of the inner boundary of that zone is derived along with a condition for its existence, which is due to plastic pressure sensitivity and Poisson's ratio.

The important case of a fully incompressible solid (where both elastic and plastic branches do not admit volume changes), which is also an extreme case of non-associativity, is discussed in some detail. The velocity profile behaves here as the inverse square of radial distance, with the deforming field extending to infinity. A quadrature type solution is given for the stresses with the elastic/perfectly-plastic characteristic. We have found that plastic pressure sensitivity causes an increase in the cavitation pressure but reduces the size of the plastic zone. These results are supported by a numerical solution for the stresses in a fully incompressible elastic/power-hardening material. In the absence of pressure sensitivity, we recover the closed form solution for the incompressible Mises material, which is valid for any hardening or softening characteristic.

Finally, we explore the material with an associated Drucker-Prager flow rule and show, with no further assumptions, that the exact field equations can be reduced to two equations for stresses. A few numerical solutions for both compressible and incompressible response reveal the coupled effect of plastic pressure sensitivity and material inertia in raising the level of cavitation pressure. By comparison, plastic compressibility appears to be much more important than elastic compressibility at low cavity expansion velocities. An elegant expansion in powers of Mach number, is derived for the cavitation pressure in a Mises solid, and shown to remain in close agreement with numerical solutions. Predictions of penetration depth, obtained from that expansion, compare favorably with available experimental data.

References

[1] Hopkins H.G.: Dynamic Expansion of Spherical Cavities in Metal. In: Sneddon, I.N., Hill, R. (Eds), Progress in Solid Mechanics **1**, North Holland, Amsterdam, 1960.

[2] Forrestal M.J., Luk V.K.: Dynamic Spherical Cavity-Expansion in a Compressible Elastic-Plastic Solid. *J. Appl. Mech* **55**, 275-279, 1988.

[3] Durban D., Fleck, N.A.: Spherical Cavity Expansion in a Drucker-Prager Solid. *J. Appl. Mech* **64**, 743-750, 1997.