AUTOMATIC 3D CRACK GROWTH SIMULATION BASED ON BOUNDARY ELEMENTS

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Summary An automatic 3D crack growth algorithm for fatigue crack propagation of arbitrary three dimensional problems with linear-elastic material behavior is presented. The stress analysis is based on the 3D Dual–BEM. An optimized evaluation of very accurate SIF's and T–stresses and the knowledge of 3D corner singularities are the basis of the proposed 3D crack growth criterion. Finally, the high level of automatization is accomplished by a sophisticated re–meshing algorithm for the update of the numerical model.

INTRODUCTION

A 3D simulation of crack growth is presented to determine the shape of 3D crack fronts as they occur during stable fatigue crack propagation as realistic as possible. It is embedded in an automatic incremental crack growth algorithm for arbitrary three dimensional problems with linear-elastic material behavior. Due to the non–linearity of crack growth, the incremental procedure mainly includes the numerical solution of the current boundary value problem and an algorithm controlling the 3D crack growth automatically.

The boundary value problem of cracked bodies is solved by a powerful tool for stress concentration problems – the boundary element method (BEM). Furthermore, the boundary–only formulation offers the advantage of an easy update of the numerical model compared to volume oriented methods.

The 3D crack growth algorithm contains the determination of an optimized new crack geometry based on a suitable 3D crack growth criterion: a predictor–corrector procedure ensuring a bounded energy release rate along the whole crack front. Finally, the numerical model has to be updated for the next increment. In case of surface breaking cracks the outer boundary needs also to be re–meshed. Therefore, an automatic local re–meshing algorithm capable of handling arbitrary smooth surfaces, which are discretized by quadrilaterals, is presented.

STRESS ANALYSIS

The boundary value problem as shown in Fig. 1 is solved by a special formulation of the BEM – the 3D Dual Discontinuity Method (DDM) – which is used in case of cracked specimens [3].

The domain $\Omega \in \mathbb{R}^3$ is homogeneous and isotropic with linear elastic material behavior. Ω is bounded by the normal boundary Γ^n and the coincident crack surfaces Γ^c and $\Gamma^{\bar{c}}$. Neumann ($\bar{\boldsymbol{t}}(\boldsymbol{x})$) and Dirichlet ($\bar{\boldsymbol{u}}(\boldsymbol{x})$) boundary conditions are prescribed along the boundary Γ.

The relevant displacement boundary integral equations (BIE) are evaluated in the framework of a collocation method on Γ^n and only on one crack surface. To separate the coincident crack surfaces, the hypersingular traction BIE is additionally applied on the remaining crack surface. Finally, the introduction of displacement and traction discontinuities along the crack as new variables leads to the DDM. The advantage of this formulation is a substantially reduced system of equations, due to the elimination of one crack surface.

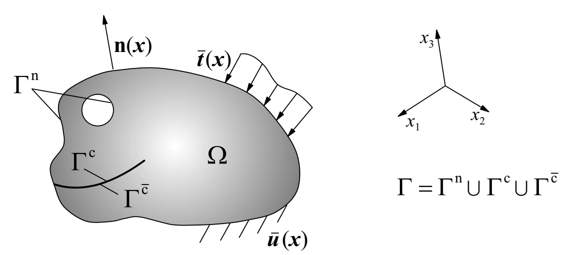

Figure 1. Sketch of the boundary value problem

A special advantage of this procedure in case of the simulation of 3D crack growth is, that it can be used within a single subregion. Especially in the area of high stress concentrations - ahead of the crack front - no discretization is needed.

3D CRACK GROWTH ALGORITHM

The three steps of the crack growth algorithm are indicated in Fig. 2. Firstly, the stress intensity factors (SIF's) and the T-stresses are evaluated very precisely along the crack front by an extrapolation method via a regression analysis [2] from the numerical crack near-field. For every crack front node a new position is determined by the evaluation of a suitable crack growth criterion. It includes the determination of the kink angle and the crack extension. With these two quantities the shape and position of the new crack front can be defined. Then, the gap between the old and new crack front has to be closed during the update of the numerical model. There are two options for this closing procedure. Depending on the amount of crack extension a new row of elements can be inserted or the old crack front nodes are moved towards the new crack front as shown in Fig. 2. However, special attention is needed in case of surface breaking cracks, as the normal boundary Γ^n has also to be considered. To ensure an optimized mesh an automatic local re-meshing procedure is needed. Subsequently, the numerical model for the next increment is generated in a fully automatic way.

Crack growth criterion

A predictor–corrector procedure concerning the crack extension with an implicit correction of the kink angle will be presented.

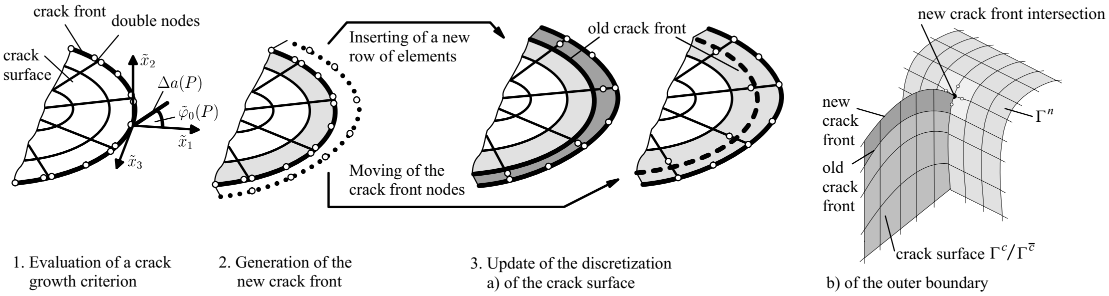

Figure 2. Basic principle of the crack growth algorithm

The kink angle is predicted by the MTS–criterion providing an angle of a differential crack extension as a function of K_{I} and K_{II}. Thus, the tangent to the deflecting crack path related to the current crack front is described. Depending on the finite incremental crack extension an error occurs. Therefore, the MTS–criterion is extended by including the T-stress T_{I} to correct the kink angle. Then, the kink angle additionally depends on T_{I} and on the finite crack extension. Moreover, this concept is expanded following the idea, that the crack growth angle is perpendicular to the maximum principal stress on an imaginary cylindrical sphere around the crack front [1]. It leads to a kink angle dependent on all three SIF's, even K_{III}. In this case, the introduction of all T-stresses leads to a kink angle which additionally depends on the Poisson's ratio and on the crack extension.

For the prediction of the crack extension a user–defined incremental length Δa_0 is specified. This length is assigned either to the maximum or average value of an appropriate fracture mechanics parameter (e.g. the energy release rate G) and is distributed linearly along the crack front. Alternatively, an exponential distribution based on the Paris Law is performed.

The shape of the crack front and therefore the resulting crack path depends on Δa_0. Thus, a corrector step is introduced to improve the shape of the crack front. It is assumed that the energy release rate controls the crack growth and the crack front will propagate in a way that G is constantly distributed along the crack front. Moreover, a valid square-root singularity is assumed along the whole crack front, especially in the vicinity of the crack front intersection with the free surface. This is ensured by an analysis of 3D corner singularities in this vicinity followed by an appropriate adjustment of the crack front angle.

Local re-meshing on Γ^n

During the update process of the discretization, the free surface must also be considered in case of surface breaking cracks. An automatic local re-meshing procedure incorporating the requirements – inserting the new crack front intersection at an arbitrary point and guaranteeing an optimal mesh quality – is designed. This procedure is based on the current discretization and consists of the steps: 1) identification of the local re-meshing patch, 2) triangulation, 3) insertion of the new crack front intersection, 4) optimization of the triangulation, 5) generation of quadrilateral elements and 6) smoothing of the new discretization.

Primarily, the local area in which the discretization needs to be optimized, has to be identified. This area (patch) is meshed generally with quadrilaterals and triangles. Therefore, a complete triangulation is performed to have a common basis for all forthcoming mesh modifications. Afterwards, the new crack increment will be inserted into the triangulation. As the triangulation is the basis for the final generation of quadrilaterals, the triangular mesh has to be optimized. Finally, the newly created quadrilaterals are smoothed.

NUMERICAL EXAMPLE

The efficiency and capability of the proposed procedure will be shown by a complex example. It is a three point bending specimen with a slanted crack. Due to the orientation of the initial crack a complex stress state including all fracture modes is caused. Consequently, the crack is twisting towards the center plane. The geometrical situation at the crack front intersection influencing the 3D singularities is changing permanently. Finally, the curved crack path is a real challenge concerning the re–meshing of Γ^n.

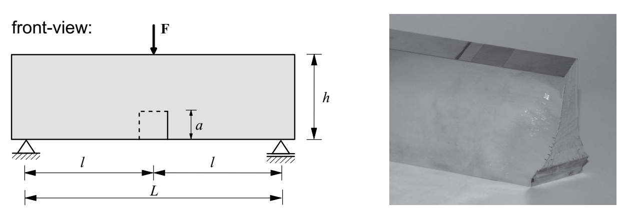

Figure 3. Three point bending specimen with a slanted crack

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