INTERMITTENT DISTRIBUTION OF HEAVY INERTIAL PARTICLES IN TURBULENT FLOWS

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Summary The phenomenon of preferential concentration of inertial particles is studied by following lagrangian trajectories. Elementary properties of the coarse-grained distribution of heavy particles in simple turbulent flows are investigated by direct numerical simulations. In the small Stokes number case, we compute the coarse-grained particle distribution, $\overline{n}_r$, and we demonstrate that the second moment $\langle \overline{n}_r^2 \rangle$ behaves as an approximate power law : $\langle \overline{n}_r^2 \rangle \sim r^{\alpha}$. The dependence of the exponent α as a function of the Reynolds and of the Stokes number is studied in the small Stokes number limit. Our results show a strong dependence of the level of fluctuation of the particle distribution as a function of the Reynolds number.

INTRODUCTION

Inertial particles advected by a turbulent flow are known to have very inhomogeneous distributions [1]. In clouds, the clustering of water droplets is known to have several important consequences [2],in particular in enhancing the collision rate between droplets, thus dramatically accelerating the formation of rain [3]. The origin of this effect is due to particle inertia : particles are advected by an effective velocity field $\mathbf{v}$, which is *compressible* ($\nabla \cdot \mathbf{v} \neq 0$) even when the surrounding fluid motion, $\mathbf{u}$ is incompressible ($\nabla \cdot \mathbf{u} = 0$). This effect is often refered to as preferential concentration.
In the limit of weak particle concentration, the density of particles, n, obeys :

$$\partial_t n + \nabla \cdot (\mathbf{v} n) = 0 \tag{1}$$

In an incompressible turbulent flow, with a Kolmogorov scale η, the divergence of the particles' velocity field $\mathbf{v}$ acts as a source term of density fluctuations at scales $\sim \eta$ [1]. As fluid elements get squashed by turbulence, the particle distribution reaches finer and finer scales, down to a cutoff length scale, r_d, determined by the diffusive process and/or by the size of the inertial particles. The clustering of particles is characterized here by the coarse-grained density of particles over a volume of size r, $\overline{n}_r$. This quantity can be expressed in terms of the properties of the trajectories of particles in the flow, conditioned by the stretching occuring along the trajectory [1, 3].
Using direct numerical simulations (DNS) of turbulent flows, we compute the properties of the second moment of the fluctuating quantity $\overline{n}_r$: $\langle \overline{n}_r^2 \rangle$ and its dependence as a function of the parameters in the problems, such as the Reynolds number, R_λ, the Stokes number, St, and size, r. A power law dependence is found numerically : $\langle \overline{n}_r^2 \rangle \propto r^{\alpha}$. In the limited range ($R_\lambda \leq 130$) studied here, a strong variation of the exponent is found.

LAGRANGIAN APPROACH

Inertial particles of density ρ_0, of radius a, evolve in a fluid of density ρ and of viscosity ν subject to an incompressible turbulent motion, described by an eulerian velocity field $\mathbf{u}(\mathbf{x}, t)$. In the low concentration case, particles can be considered as purely passive. Introducing $\beta = 3\rho/(\rho + 2\rho_0)$ and the Stokes time, $\tau_s = a^2/3\nu\beta$, the velocity $\mathbf{v}$ of inertial particles satisfies [2] :

$$\frac{d\mathbf{v}}{dt} - \beta \frac{d\mathbf{u}}{dt} = (\mathbf{u} - \mathbf{v})/\tau_s \tag{2}$$

We restrict ourselves here to the case where the time τ_s is short compared to the Kolmogorov time scale, $\tau_K = \langle \omega^2 \rangle^{1/2}$, so the Stokes number, $St \equiv \tau_s/\tau_K$ is small. The velocity is well approximated by $\mathbf{v} = \mathbf{u} - \tau_s \frac{d\mathbf{u}}{dt}$, where $\frac{d\mathbf{u}}{dt}$ is the lagrangian derivative of the velocity field : $\frac{d\mathbf{u}}{dt} \equiv \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u}$. The velocity field $\mathbf{v}$ is *compressible* : $\nabla \cdot \mathbf{v} = -\tau_s \nabla \cdot (\mathbf{u} \cdot \nabla)\mathbf{u} = -\tau_s tr(m^2)$, where m is the velocity derivative tensor : $m_{ij} = (\partial u_j / \partial x_i)$. The source of concentration fluctuations is related to the compressibility of the velocity field $\mathbf{v}$; it acts therefore most strongly at the Kolmogorov size, η [1, 3]. To estimate the coarse-grained distribution of particles over a size r, $\overline{n}_r$, at location $\mathbf{x}$ and time t, one has to trace back in time to the previous time $-t_r$, and to the location $\mathbf{x}_r$, such that the trajectory starting at $-t_r$, and location $\mathbf{x}_r$ ends up at $t = 0$ at location $\mathbf{x}$. The flow maps a small volume of characteristic size r at $t = 0$ to a volume whose largest scale is η at time $t = -t_r$. The intuitive picture is that particle concentration fluctuations accumulate at a scale r as long as it takes to expand the volume to a size $\sim \eta$ as the trajectory evolves backwards in time. The coarse-grained particle concentration is [1] :

$$\overline{n}_r(\mathbf{x}, t) = n_0 \exp\Big(\tau_s \int_{-t_r}^{0} tr(m^2)(t')dt' \Big) \tag{3}$$

The statistical weight associated with each trajectory is $\propto 1/n$. As a consequence, the second moment of the distribution of $\overline{n}_r$ is simply $\langle \overline{n}_r^2 \rangle_E = \langle n_0 \exp\Big(\tau_s \int_{-t_r}^{0} tr(m^2)(t')dt' \Big) \rangle_{Lag}$, where $\langle ... \rangle_E$ ($\langle ... \rangle_{Lag}$) refers to eulerian average (average over a set of trajectories).
In the small Stokes number limit, the difference $|\mathbf{v} - \mathbf{u}| \sim St|\mathbf{u}| << |\mathbf{u}|$ is small. We thus simply follow the compression along lagrangian trajectories of the flow to obtain an estimate of the quantity $\langle \overline{n}_r^2 \rangle$.

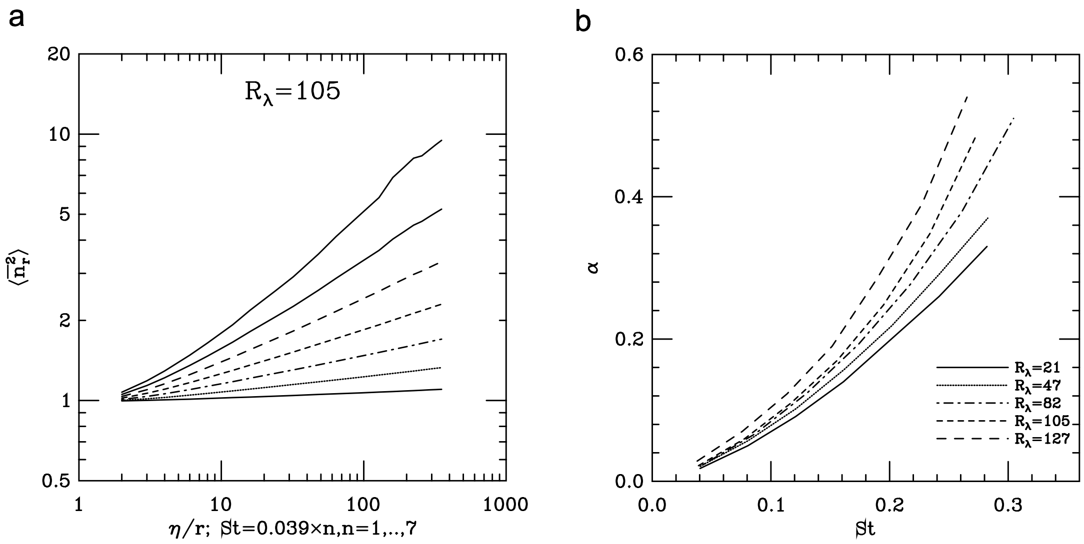

Figure 1. The variance of the concentration fluctuation $\langle \overline{n}_r^2 \rangle$ as a function of r, at $R_\lambda = 105$ and for $St = n \times 0.039$ (1a), and the exponent α as a function of St for several values of R_λ (1b).

NUMERICAL METHODS

A turbulent velocity field is generated by standard DNS methods [4]. At a given time, a set of N_l lagrangian trajectories is released. Along with the locations and velocities of the particles, we follow the tensor W describing the deformation of small (infinitesimal) fluid elements, as well as its inverse : $\frac{dW_{ij}}{dt} = m_{ik} W_{kj}$ and $\frac{dW_{ij}^{-1}}{dt} = -W_{ik}^{-1} m_{kj}$. The compression between time $-t_r$ and 0 of a fluid parcel of size η to a small scale r implies that the norm of W^{-1} grows by a factor (η/r) during this lapse of time. By monitoring the tensor W^{-1}, we are therefore able to determine the final location $\mathbf{x}$ of a trajectory, starting at $\mathbf{x}_0$, such that the compression of a fluid element between the initial $t = 0$ and the final time $t = t_r$ is equal to a given value (η/r). This allows us to determine the quantities X_r and $\overline{n}_r$ without having to integrate the equation of motion for the fluid particles backwards in time.

The number of grid points was varied in the range $32^3 - 256^3$. The product $k_{max}\eta$, where k_{max} is the highest wavenumber represented in the simulation is maintained in the range $1.4 \leq k_{max}\eta \leq 2$, thus assuring that the small scales are adequately resolved. The Reynolds number is varied in the range $20 \leq R_\lambda \leq 130$. Lagrangian trajectories are followed using the algorithm developped in the tensor W and W^{-1} are integrated according to the method introduced in [5].

SCALE DEPENDENCE OF THE COARSE-GRAINED DENSITY FLUCTUATION

Fig.1a shows a typical example of the r-dependence of $\overline{n}_r^2$ as a function of r, at $R_\lambda = 105$, and at a value of the Stokes number $St = 0.0390 \times n$, $n = 1, ..., 7$. Away from the values of r close to $\eta/r = 1$, $\langle \overline{n}_r^2 \rangle$ behaves almost as a power law, as revealed by the straight lines seen on the log-log plot. The curves however show a slight convexity.

The exponent can be computed directly from the numerical data by finite differences. Our numerical results demonstrate that a power law dependence is a very good approximation of the experimental data. The exponent was systematically measured at a value of $\eta/r = 200$. Fig.1b shows the exponents as a function of the Stokes numbers for several values of the Reynolds numbers. Remarkably, it is found that the exponent α grows strongly when the Reynolds number increases.

DISCUSSION

We have analysed the phenomenon of preferential concentrations by following, along individual particle trajectories, the quantity $tr(m^2)$. Our results show that the effect of preferential concentration is strongly dependent on the properties of the turbulence. The very small scale, intermittent structures of turbulence, such as vortex tubes, thus have a direct and important impact on the distribution of heavy particles in turbulent flows.

References

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