MOBILITY OF MEMBRANE-TRAPPED PARTICLES: PROTRUSION INTO THE SURROUNDING FLUID

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Short summary: Rheological and transport studies of model thin films and membranes, often inspired by biological systems, make use of translational or rotational motion or diffusion of particles trapped in the surface film. Here, the mobility of a disk-shaped particle, trapped in a Newtonian surface film and bounded on one side by a viscous Newtonian fluid, is determined for when the particle protrudes into the subphase.

DYNAMICS OF MEMBRANE-TRAPPED PARTICLES

The translation of particles along membranes and interfaces is of interest because it is a model system for describing basic features of interfacial hydrodynamics, it is important in cellular signalling in biology and (Alberts et al. 1994), and it is potentially applicable for understanding dynamics of structured materials. The theoretical investigation of such motions of membrane-trapped particles began with the study of the Brownian motion of a thin *disk-shaped* object at a flat viscous interface bounded on one or both sides by a viscous Newtonian fluid (Saffman 1976; Saffman & Delbruck 1975). The case of a sphere at a surfactant-coated interface was treated numerically by Danov et al. (1995). The hydrodynamics of these two different shapes differ since the protrusion of the sphere into the subphase changes the relative contributions of the surface and subphase resistances. Recent experiments by Sickert & Rondelez (2003) with spherical particles as tracers in monolayers in the liquid-expanded phase motivated our work, which treats protrusions intermediate between the previously studied cases of a thin disk (no protrusion) and a sphere.

The mathematical problem considered originally by Saffman was that of a flat, disk-shaped particle of radius R that spans the thickness of the fluid layer (a monolayer or bilayer) at a fluid-air or fluid-fluid interface (see figure 1). The object is assumed to move at a constant speed U and the force F on the object is calculated in the low Reynolds number limit. The ratio F/U is the particle resistivity, which is related to the measurable translational diffusivity D by a Stokes-Einstein relation, $D=k_BT/(F/U)$. The resistance comes from both the sub- and surface phases and the dynamics are conveniently characterized in terms of a dimensionless parameter $L=Rm/hm_m$, where m is the fluid viscosity, m_m is the viscosity of the membrane fluid, and h is the thickness of the surface layer. The commonly discussed surface viscosity $m_s=m_mh$. Saffman was interested in small proteins so $L<<1$. On the other hand, frequently membrane-trapped objects (e.g. phase separated domains) are large in which case $L>>1$ (e.g. Klinger \& McConnell 1993; Sickert & Rondelez 2003).

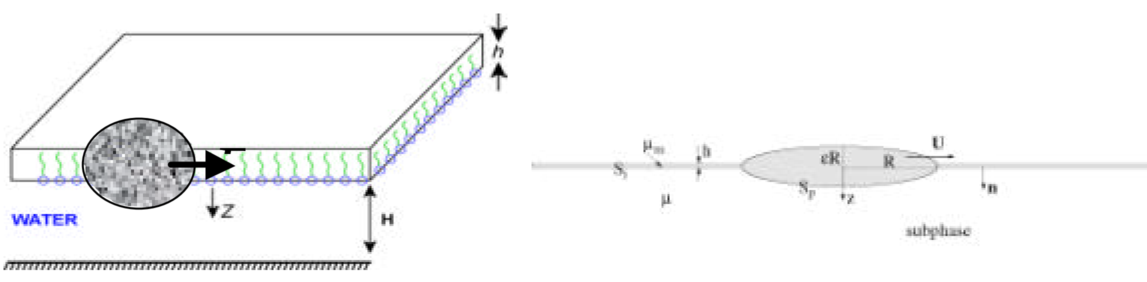

Figure 1: Left – A sphere straddling an interface and translating along the interface. Right – The mathematical characterization of an oblate ellipsoid, with aspect ratio e, straddling the interface.

THE INFLUENCE OF PROTRUSION INTO THE SUBPHASE

We ask ``what if the surface motion is generated by an object that protrudes into the subphase?'' The protrusion causes the flow to be fully three dimensional, unlike the treatment of disk-shaped objects (Saffman 1975; Hughes, Pailthorpe & White 1981; Stone & McConnell 1995; Stone & Ajdari 1998). Danov et al. (1995) considered numerically the case of a sphere straddling a surfactant-coated interface and for the limit L >>1, obtained a drag that is the same as if the spherical particle were translating in a unbounded Newtonian fluid, i.e. $F/F^{Stokes} = 1$; the influence of the incompressible surface film was negligible. This limiting result is consistent with that given above for a flat disk if the velocity gradients associated with the three-dimensional flow dominate any influences associated with the surface film.

OUR CONTRIBUTION

We consider the intermediate case of a disk-shaped particle that protrudes into the subphase and seek to quantify the variation of the force as the degree of protrusion changes. We assume the shape is a nearly flat oblate spheroid with degree of oblateness indicated by e; $e=0$ is a flat disk and $e=1$ is a sphere. We first illustrate for an oblate spheroid translating in an unbounded fluid that just one term of an expansion in e is an excellent agreement for all $0<e<1$with an exact result (Happel & Brenner 1983). Hence, for the membrane-bound particle problem we expect, or at least suggest, that just one term of a perturbation expansion in ε should similarly be expected to provide a good description.

Our calculation uses a domain perturbation to relate the oblate spheroid, for which results are unknown, to a disk shape, for which results are known. Hence, we expand the velocity field as $\boldsymbol{u}(\boldsymbol{r};\varepsilon)=\boldsymbol{u}^{(0)}(\boldsymbol{r})+\varepsilon\boldsymbol{u}^{(1)}(\boldsymbol{r})+\cdots$ with a similar equation for the force $\boldsymbol{F}$ on the membrane-trapped object; the leading-order velocity field $\boldsymbol{u}^{(0)}(\boldsymbol{r})$ corresponds to that of a thin-disk. Using the Reciprocal Theorem from low-Reynolds-number hydrodynamics, suitably modified to handle the interfacial flow and boundary conditions, and denoting the particle shape by $f(r)$, we arrive at the correction to the force $\boldsymbol{F}^{(1)}$,

$$\boldsymbol{F}^{(1)}\cdot\boldsymbol{U}=-2\int_{r=0}^{1}\int_{\theta=0}^{2\pi}f(r)\left(\frac{\partial\boldsymbol{u}^{(0)}}{\partial z}\right)^{2}r\,\mathrm{d}\theta\,\mathrm{d}r\,,$$

which can be evaluated numerically using known results for the leading-order velocity field. Finally, we discuss how these results may be useful for interpreting recent experiments by Sickert & Rondelez (2003).

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