ON THE THEORY FOR SUBSONIC, TRANSONIC AND SUPERSONIC FLOWS IN WATER WITH SUPERCAVITATION

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Summary Theoretical consideration are performed to improve further understanding of compressibility effects as a whole in the new field of hydrodynamics at super high speeds in the range of the speed of sound of water which is order of c~1500m/s.

For very high speed in water the body motion is realized in a vapor filled cavity, created by a small blunt nose part of the body, the so called cavitator. Under these conditions most part of the body is separated from the liquid fluid, which implies the possibility to avoid considerable viscous losses and achieve a very low total drag, comparable to that of high speed motion in air. Up to now classical supercavitation modelling is based mainly on the incompressibility assumption and can be found in number of well known books, e.g. of G Birchoff and E. Zarantonello [1], M. Gurevich [2]. Axisymmetric flow can be illustrated by a simple heuristic model which follows from the Slender Body Theory (SBT). Moving through a fluid at rest, the cavitator pushes fluid aside and its momentum is converted into energy of a practically radial flow in every lateral section of the cavity. Further, a practically independent expansion and compression of lateral cavity sections occur in interaction with the surrounding pressure p_∞ and pressure in the cavity $p_c \sim 0$. Minimum drag coefficients are achieved by establishing the maximal slenderness of the cavity. One of the most effective technique for prediction of minimum drag is the application of simple heuristic models, together with integral conservation laws and combined with the perturbation theory. In the past, this approach gave the possibility yet for 40-50 years to estimate shape and main scaling of cavities, e.g. by H. Reichardt, A. May , G. Logvinovich , P. Garabedian and others. Considerable advances have been achieved by M. Tulin and his 2-D linear theory for supercavitation, which stimulated the development of a linearized axisymmetric theory using the SBT and the Matched Asymptotic Expansion Method (MAEM) [3-4]. Important results here were obtained by C.C. Grigorian, Yu. L. Yakimov, A.G. Petrov too. The base of the linearized theory is the integro-differential equations (IDE) for slender axisymmetric cavities and it's solution with help of asymptotic expansions with the slenderness parameter δ. The most important configuration is the singular case for small cavitators including a disc. The solution is separated into the 3 zones: The inner near cavitator solution (nonlinear for disc), the intermediate solution on the base of known Gurevich - Levinson asymptotic development at infinity and the solution for the outer field as ellipsoidal cavity perturbation problem. With the help of matching and by using the M. Van Dyke additive rule suitable solutions are found. The (IDE) for $M = 0$ has the same form as that for $M < 1$ and for $M > 1$ it has the same structure, which gives the possibility to apply this approach for the theory for $M < 1$ and $M > 1$ too. Despite of essential achievements the situation in the field of super high speeds in water a as whole is far from ideal. Note, here one of the first experiments for $M > 1$ [5] at speeds of motion till $1000\,\mathrm{m/s}$ has been performed by Yu. Yakimov, speeds till $1200-1300\ \mathrm{m/s}$ - in [6], not high $M > 1$ have been investigated in [7]. Small scale experiments have been performed by: Yu. Bivin - Yu Gluchov - Yu. Permiakov, C. Voidneck, M. Schaffar too. All experiments are for the natural pressure of $1\ \mathrm{bar}$ only. Nonlinear numerical calculations in case of cone and disc for $M < 1$, $M > 1$ are obtained in [8], a numerical-analytical approach for $M < 1$ is developed in [9]. Results of linearized approaches for $M < 1$, $M > 1$: linear 2-D theory in [10], axisymmetric linearized theory in [11,12], 2-D plane theory in [14]. Numerical calculations have been presented by A. Terentiev - A. Chechnev, generalization of the model for top high speed - by R. Saurel, J.P Cocchi, P.B. Butler, and an attempt to predict finite cavities in the regime of not large $M > 1$ - by A. Vasin. The existing theoretical models have been developed for total pressures till $25000-30000\ \mathrm{bar}$, by applying Tait's equation of state for water in adiabatic form, together with the compressible Bernoulli equation and the know dependence for the speed of sound:

$$\text{a)}\ \frac{P+\mathbf{B}}{\rho^n} = \frac{P_\infty+\mathbf{B}}{\rho_\infty^n}, \qquad \text{b)}\ c^2 = \frac{dP}{d\rho} = \frac{n(P+\mathbf{B})}{\rho}, \qquad \text{c)}\ \frac{n}{n-1}\frac{P+\mathbf{B}}{\rho} + \frac{(U_\infty+u)^2+v^2}{2} = \frac{n}{n-1}\frac{P_\infty+\mathbf{B}}{\rho_\infty} + \frac{U_\infty^2}{2}, \qquad (1)$$

$\mathbf{B} = 3045\,\mathbf{kg}/\mathrm{cm}^2$, $n = 7{,}15$, p, p_∞; ρ, ρ_∞ pressure, density in the flow and at infinity, respectively, u, v components of perturbation speed, U_∞ speed at infinity. (IDEs) for the slender axisymmetric cavities $r = R(x)$ on the base of SBT are:

$$M<1:\quad \frac{1}{2R^2}\left(\frac{dR^2}{dx}\right)^2 + \frac{d^2R^2}{dx^2}\ln\frac{m^2R^2}{4x(L-x)} - \int_0^{x_o}\frac{\frac{d^2r_1^2}{dx^2}\Big|_{x=x_1} - \frac{d^2R^2}{dx^2}}{|x_1-x|}dx_1 - \int_{x_o}^{L}\frac{\frac{d^2R^2}{dx^2}\Big|_{x=x_1} - \frac{d^2R^2}{dx^2}}{|x_1-x|}dx_1 - \frac{\frac{dr_1^2}{dx}\Big|_{x=0}}{x} + \frac{\frac{dR^2}{dx}\Big|_{x=L}}{L-x} = 2\sigma_\infty$$

$$M>1:\quad \frac{1}{2R^2}\left(\frac{dR^2}{dx}\right)^2 + \frac{d^2R^2}{dx^2}\ln\frac{m^2R^2}{4x^2} - 2\int_0^{x_o}\frac{\frac{d^2r_1^2}{dx^2}\Big|_{x=x_1} - \frac{d^2R^2}{dx^2}}{|x_1-x|}dx_1 - 2\int_{x_o}^{x}\frac{\frac{d^2R^2}{dx^2}\Big|_{x=x_1} - \frac{d^2R^2}{dx^2}}{|x_1-x|}dx_1 - 2\frac{\frac{dr_1^2}{dx}\Big|_{x=0}}{x} = 2\sigma_\infty \qquad (2)$$

$x = x_o$ -separation section, $\sigma_\infty = 2(p_\infty - p_c)/\rho_\infty U_\infty^2$ - cavitation number, $m = \sqrt{|M_\infty^2 - 1|}$, $r = r_1(x)$ - a slender cavitator form. Interstitial (ODE) are extracted from (IDE) and its intermediate asymptotics Eqs.3, and from asymptotic solutions for $\sigma = 0$ in the particular in case of a disc for $M<1$ Eq. 4a , and for a slender cone for $M>1$ Eq. 4b are obtained:

$$M<1:\quad R^2 = \frac{2\sqrt{c_{do}}\,x}{\left[\ln\left(x^2/R^2\right)\right]^{0.5}}\left[1 - \frac{1}{2}\frac{\ln\left(4/em^2\right)}{\ln\left(x^2/R^2\right)} + \ldots\right] \sim \frac{x}{(\ln x)^{0.5}};\qquad M>1:\quad R^2 = \frac{K_s}{(\ln x^2/R^2)^{3/2}}\left[1 + \frac{3}{2}\frac{\ln m^2/4}{\ln x^2/R^2}\right] \sim \frac{x}{(\ln x)^{3/2}} \tag{3}$$

$$\text{a)}\quad R^2 = 1 + \frac{2\sqrt{(c_{do})}\;x}{\sqrt{\ln\left\{\dfrac{4(\Delta + x/\sqrt{e})^2/m^2}{[1 + 2\sqrt{(c_{do}}\;x/\sqrt{\ln(4\Delta^2 + x)}]}\right\}}},\qquad \begin{matrix} R_n = 1 \\ \Delta = 0.5\left[\sqrt{c_{do}} + 1/\sqrt{c_{do}}\right] \end{matrix};\quad \text{b)}\quad R^2 = \varepsilon^2\left[2x\left(\frac{\ln 1/\varepsilon^2}{\ln x/\varepsilon^2}\right)^{3/2} - 1\right]_{x\to\infty} \sim \frac{x}{(\ln x)^{3/2}} \tag{4}$$

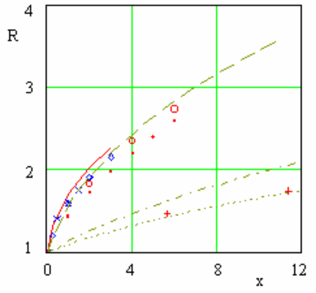

Fig. 1 Front part of cavities, $\sigma = 0$
— — — disc $M = 0$ solution Eq.4,
——— numerical calculation L. Guzevsky ,
◇◇◇◇, × × × × experiment G. Logvinivich
— - — cone $\gamma = 10^o$ solution for $M = 0$
o o o o . • • • • disc $M = 1$, $M = 2$
nonlinear numerical calculation [7]
- - - - - - cone $\gamma = 10^o$, $M > 1$ solution Eq. 4
+ + + cone $\gamma = 10^o$, $M = 2$ [7]

Here $R_n = 1$ is the disc radius, $\varepsilon = \tan\gamma$, γ - cone semi-angle, c_{do} - cavitator drag coefficient for $\sigma = 0$, K_s - asymptotic constant for $M>1$. The linearized second order theory [11-12] predicts not large compressibility influence at $M<1$, experiments [6] confirm Eqs. 3, 4 until $M \sim 0.6\text{-}0.8$. In case of cavities behind slender cavitators for $M>1$ the compressibility influence is not large, but if the cavity become essentially large as compared to the cavitator this influence can become very important. For $M<1$ the solutions of the Eqs. 3, 4 express the energy conservation law, for $M>1$ - a considerable wave loss appears depended on the cavitator form. For $M>1$ the asymptotic behavior of Eq. 3 is confirmed by experiments [7], even for very not high $M>1$, solutions of the type of Eq. 4 for a cone are confirmed by numerical calculations [8]. As result the shape of the front part of cavities at $M>1$ can be considerable more narrow as compared with $M<1$. The results of the prediction of the front part of the cavities are demonstrated by Fig. 1. The compressibility influence on the aspect ratio of finite cavities is not large and is estimated by the known dependence $\lambda = \sqrt{(\ln 1.5m^2/\sigma)/\sigma}$. At the same time the coefficient $k \sim 1$ in the known formula for the maximal radius $R_k = R_n\sqrt{c_d/k\sigma}$ at $M<1$ - in case of $M>1$ can be essentially more than 1 and accordingly the size of the finite cavities can be considerable more small as compared with the $M<1$ case. Thanks to the very high adiabatic coefficient $n = 7{,}15$ a very vide transonic Mach number regime in water with mixed flow of $M<1$ and $M>1$ is discovered, here transonic equation is applied:

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \left[\left(1 - M_\infty^2\right) - \frac{(n+1)M_\infty^2}{U_\infty}\frac{\partial\varphi}{\partial x}\right]\frac{\partial^2\varphi}{\partial x^2} = 0,$$

what gives the possibility to estimate the aspect ratio of transonic cavities and the drag coefficient of slender cavitators [12]. At present the most important topic is the investigation of the transonic supercavitating flows and making additional special experiments for verifications of the theory.

References

[1] Birkhoff G., Zarantonello E.: Jets, Wakes and Cavities. New York, Academic Press,1957.
[2] Gurevich M. I.: Theory of Jets of Ideal Fluid. Moscow, Nauka. 1978.
[3] Serebryakov V.V.:Asymptotic Solution of the Problem on Slender Axisymmetric Cavity", *J. DAN of Ukranian SSR, ser. A* , 12, 1973.
[4] Logvinovich G. , Serebryakov V.: On Methods of Calculation of Slender Axisymmetric Cavities Shape. *J. Hydromechanics* 32, 1975.
[5] Howard Mc-Millen J., Newton Harwey E.: A Spark Shadow Graphic Study of Body Waves in Water". J. Applied Physics, 17, 7, 1946.
[6] Savchenko Yu. N., Semenenko V. N.., Serebryakov V. V.: The Experimental Investigations of Supercavitation in Subsonic Flow. *J. DAN of Ukraine* 2 , 1993.
[7] Kirschner I. N.: Supercavitating Projectile Experiments at Supersonic Speeds. Agard-R-827, 1998.
[8] Aleve G. A.: Separating Flow of Circle Cone by Transonic flow of Water. *J. Trans. of AS USSR, MFG* :1, 1983.
[9] Zigangareva L. M. , Kiselev O. M.: Subsonic Flow of Compressible Fluid for Small Cavitation Numbers. *Trans RAN, MFG* 4, 1998.
[10] Nishiyama T. and Khan O.: Compressibility effects upon cavitation in High-Speed Liquid Flow (Transonic and Supersonic Liquid Flows). *Bulletin of the Japanese Society of Mechanical Engineers* 24, 190, 1981.
[11] Serebryakov V.V.: Asymptotic Solutions of Axisymmetric Problem for Sub- and Supersonic Separate Flows in Water under Zero Cavitation Number. *J. DAN of Ukraine* 9, 1992.
[12] Serebryakov V.V., Schnerr: G.H.: Some Problems of Hydrodynamics for Sub-, Trans- Supersonic Motion in Water with Supercavitation. *Proc. of 4-th Intern. Symposium on Cavitation, Osaka, Japan*, 2003, web site **http://flow.me.es.osaka-u.ac.jp/cav2003**.
[13] Mayboroda A. N.: Planing for Sub- and Supersonic Speeds. Intern. Sci. School “High Speed Hydrodynamics”, Chebocsary, Russia, 2002.