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NEW RESULTS OF STRUCTURAL OPTIMIZATION FOR POST-BUCKLING BEHAVIOUR

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Summary Recently the new concept of structural optimization against instability has been proposed. The post-buckling analysis is included directly in the optimization problem and the so-called modified design is formulated. As a result, the optimal structure with stable behaviour after buckling is obtained. Many modified design problems for structures exposed to elastic instability have been already formulated and solved. The paper presents general approach and new results.

INTRODUCTORY REMARKS

Maximization of instability load for a prescribed volume of a designed element is the standard problem of optimization under stability constraints. The analysis of nonlinear post-buckling behaviour and the influence of imperfections are, in general, not included in such a standard formulation and therefore the important information about behaviour of a designed element after buckling is not provided. Very often the standard optimal structure represents unstable post-buckling behaviour. This phenomenon is dangerous in engineering practice since the applicability of such an unstable optimal structure may be questionable. It indicates that a modification of such a standard optimization is necessary especially from practical point of view. The concept of modified optimization including constraints imposed on stability of post-buckling behaviour has been proposed by Bochenek [1]. A broad classification of the modified design problems is given in [2]. Among various proposals of new design problems is optimization against instability in the large. It refers to the case when critical state does not exist and instability occurs only at finite displacements. The standard optimization problem cannot be formulated, but modified design against this form of instability can be performed. This case is discussed in the present paper. In several previous papers Bochenek considered many variants of the modified optimization and obtained effective results using purely numerical approach based on discretization and non-linear programming. Parallelly, Życzkowski [3] derived analytical formulae determining post-buckling path for columns under general behaviour of loading, including active and reactive forces. Making use of both these directions of research alows to develop analytical approach to optimization for post-buckling behaviour. We give here some details concerning that approach. It occurs that modified designs are sensitive to geometry changes in pre-buckling state. This interesting and important problem is demonstrated in this paper on selected examples: optimization of a column loaded by an attached vessel containing liquid and design of a helical spring.

NUMERICAL OPTIMIZATION AGAINST INSTABILITY IN THE LARGE

This paper undertakes the problem of modified optimization against instability in the large providing formulations and numerical solutions for selected elements. The typical deformation path of the structure that suffers from instability in the large is a curve that decreases from infinity, passes its minimum and then increases. The structure instability can be characterized by the minimum value of the deformation curve which refers to lower critical load. In what follows the optimization problem can be formulated either as maximization of lower critical loading or minimization of generalized displacement for lower critical load or maximization of generalized displacement for which unstable equilibrium begins. These formulations are presented in the Fig.1.

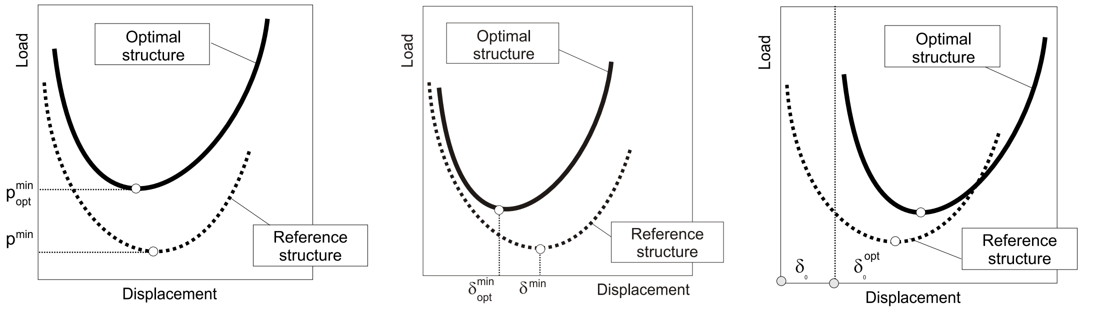

Figure 1. Maximization of lower critical loading when upper critical load is absent (left). Minimization of generalized displacement for lower critical load (middle). Maximization of generalized displacement for which unstable equilibrium begins (right).

Example of compressed and transversally loaded beam placed on the rigid foundation

To illustrate the discussion of this section a cantilever beam placed on the rigid foundation, loaded by an uniform transverse loading and compressed by an axial force, has been chosen. If the value of the transverse load is greater than zero critical state of the beam does not exist, and only for finite values of vertical displacement unstable equilibrium may be observed. Therefore the nonlinear analysis is needed to provide the system response and, what is important from optimization point of view, the design against instability in the large has to be performed.

ANALYTICAL OPTIMIZATION OF COLUMNS UNDER GENERAL BEHAVIOUR OF LOADING

Until now the modified optimal solutions have been obtained using purely numerical approach based on discretization and non-linear programming. Recently Życzkowski [3] derived analytical formulae determining post-buckling path under general behaviour of loading, including active and reactive forces. Based on that analytical approach to optimization of columns for post-buckling behaviour has been developed. Perturbation method combined with Croll's manoeuvre makes it possible to derive general formulae for the subsequent corrections of the force, and hence to analyze stability in the post-buckling range. Based on that the analytical formulae of the postbuckling constraints can be obtained. The constraint imposed on the second correction of the force allows to obtain stable post-critical behaviour of the designed structure locally that is in the vicinity of critical point. The relevant optimization problem is then expressed as an isoperimetric problem of the calculus of variations. In many cases of loading behaviour three ranges for structural parameters may be distinguished: 1) post-buckling stability both for prismatic columns and optimal columns designed for only buckling constraints, 2) post-buckling instability for optimal columns which may be changed into stability by using modified optimization, 3) post-buckling instability in any case. Analytical approach makes it possible to evaluate boundaries of these ranges. The proposed approach is illustrated by the example described in the following section.

INFLUENCE OF GEOMETRY CHANGES IN PRE-BUCKLING STATE ON MODIFIED OPTIMAL DESIGNS

Analytical optimization of a column loaded by an attached vessel containing liquid

Consider a column loaded by an attached vessel filled with a heavy liquid with specific weight γ. The vessel has the form of a symmetric triangular parallellepiped characterized by the dihedral central angle 2β and depth b. The column is loaded by the concentrated axial force.

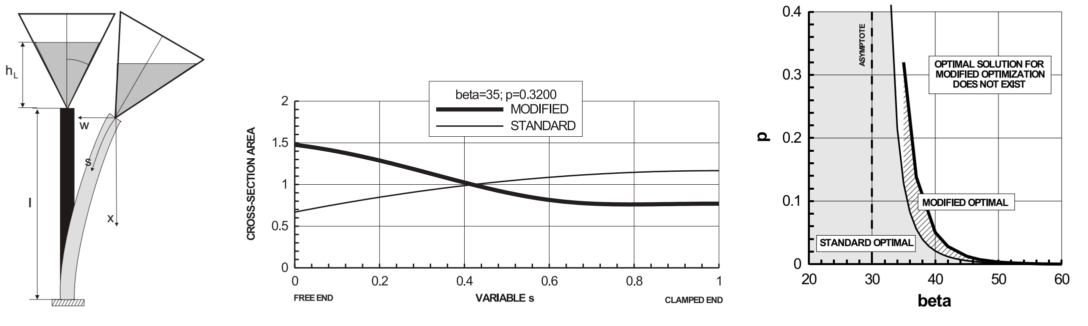

Figure 2. A column loaded by an attached vessel containing liquid (left). Standard and modified optimal column (middle). The three ranges for structural parameters (right).

Loading of the column may be controlled by any of two parameters: specific weight γ or by the volume of liquid V_l. In the first case (rather theoretical, but apparently possible e.g. if a charged liquid is subjected to an electromagnetic field with increasing intensity) we have no pre-buckling geometry changes. In the case of loading controlled by V_l pre-critical geometry changes occur and the differences in shape of standard and modified optimization are more visible.

Numerical optimization of helical springs

A helical spring is an example of a structural element, for which large displacements and changes of geometry should be taken into consideration in a precritical state. It influences the buckling load and the post-critical behaviour of a structure. While a helical spring with a constant radius and constant helix angle has usually a locally stable postcritical path, on the contrary the optimal spring with varying helix angle can show unstable behaviour after buckling. Then the standard formulation of the optimization problem should be modified and post-buckling constraints should be taken into consideration.

CONCLUSIONS

Some structures, in particular columns, optimized for minimal volume under prescribed critical loading (standard optimization) show unstable post-buckling path. Such cases, if possible, should be avoided from engineering point of view. The present paper gives both numerical and analytical treatment of the so-called modified optimization ensuring at least locally stable post-buckling path. General behaviour of loading after buckling is considered (path-dependent loading).

References

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