XXI ICTAM, 15–21 August 2004, Warsaw, Poland

EFFECT OF HORIZONTAL COMPONENT OF THE CORIOLIS FORCE ON PROPAGATION OF NEAR-INERTIAL WAVES IN THE OCEAN

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Summary Oceanic motions of scales small compared to the Earth's radius are commonly described as if the rotating Earth were locally ¤at with the horizontal component of the Coriolis force being neglected (the so-called "traditional approximation"). We show that taking into account the horizontal component of the Coriolis force changes dramatically dynamics of near-inertial waves.

A new family of sub-inertial waves, which are absent under the "traditional approximation", is found to play a crucial role: On the "non-traditional" beta-plane inertial waves propagating poleward and reaching their inertial latitude are not re¤ected at this latitude, as is the case under the "traditional approximation", but turn into subinertial waves which propagate further poleward trapped within near bottom and near surface wave-guides around the minima of the buoyancy frequency. Their horizontal and vertical scales rapidly decrease and tend to zero at a critical latitude. There is no re¤ection and, thus, inertial waves are absorbed contributing to deep ocean mixing.

Oceanic motions of scales small compared to the Earth's radius are commonly described as if the rotating Earth were locally ¤at with the horizontal component of the Coriolis force being neglected (the so-called "traditional approximation"). We show that taking into account the horizontal component of the Coriolis force changes dramatically propagation of near-inertial waves and has major implications for dynamics of deep ocean.

We begin with the standard linear equations on the non-traditional f-plane under the Boussinesq approximation, which can be reduced to a single equation for vertical velocity w

$$\nabla^2 w_{tt} + (\vec{f}\cdot\nabla)^2 w + N^2 \nabla_h^2 w = 0 \qquad (1)$$

We employ the following Cartesian frame: x (west-east), y (south-north); z (vertical, positive upward); u, v and w are the corresponding velocity components; $\vec{f} = (0, \tilde{f}, f)$, and ∇_h^2 denotes the horizontal Laplacian. We allow the buoyancy frequency N to depend on z. In the traditional approximation one would take $\tilde{f} = 0$.
For plane monochromatic ($w = W\exp(i\sigma t)$) waves travelling in the $\chi = x\cos\alpha + y\sin\alpha$ direction, we £nd

$$(N^2 - \sigma^2 + f_s^2)W_{\chi\chi} + 2ff_sW_{\chi z} - (\sigma^2 - f^2)W_{zz} = 0\,, \qquad (2)$$

where $f_s = \tilde{f}\sin\alpha$. Equation (2) is the starting point of our study.
On employing the standard boundary conditions at the surface and the bottom the substitution of $W = \psi(z)\exp i(k\chi + \delta z)$, with $\delta = -kff_s/(\sigma^2 - f^2)$ leads to the following BVP for ψ

$$\psi'' + k^2\Big[\frac{N^2(z) - \sigma^2}{\sigma^2 - f^2} + \Big(\frac{\sigma f_s}{\sigma^2 - f^2}\Big)^2\Big]\psi = 0\,, \qquad \psi(0) = \psi(H) = 0, \qquad (3)$$

Note, that from purely mathematical perspective the "non-traditional term" containing f_s, does not pose any dif£culty, as it produces merely an additive constant. However it results in the following qualitative effects. In the vicinity of the inertial frequency the solution becomes independent on $N(z)$ and can be readily found for the $n-$th mode

$$(\sigma^2 - f^2)/\sigma = \pm\frac{f_sH}{2\pi n}k, \qquad k > 0 \qquad (4)$$

In contrast to the "traditional" boundary-value problem where for all eigenmodes $\sigma^2 - f^2 > 0$, we, in addition, have another family of *sub-inertial* modes with $\sigma^2 - f^2 < 0$, which being con£ned to a narrow $O((f_s/N)^2)$ frequency range are trapped in the waveguides around minima of $N(z)$, i.e near surface and bottom of the ocean. In the longwave limit frequencies of the modes of both families tend to f with £nite group velocity $d\sigma/dk = f_sH/2\pi n$.

The importance of the new family of sub-inertial waves, which are absent under the "traditional approximation", becomes apparent if the beta effect or any kind of large scale nonuniformity is taken into account. On the "non-traditional" beta-plane near inertial waves are described by the equation

$$(N(z)^2 - \sigma^2 + \tilde{f}^2)W_{yy} + 2f\tilde{f}W_{yz} - (\sigma^2 - f^2 - 2f\beta y)W_{zz} = 0\,, \quad W(0) = W(H) = 0 \qquad (5)$$

which is the starting point of further analysis.
Under the "traditional approximation" inertial waves propagating poleward slow down approaching their inertial latitude and then are totally re¤ected back. On the "non-traditional" beta-plane the picture of wave evolution is entirely different: near inertial waves propagating poleward do not slow down on reaching the inertial latitude, they pass it with £nite group

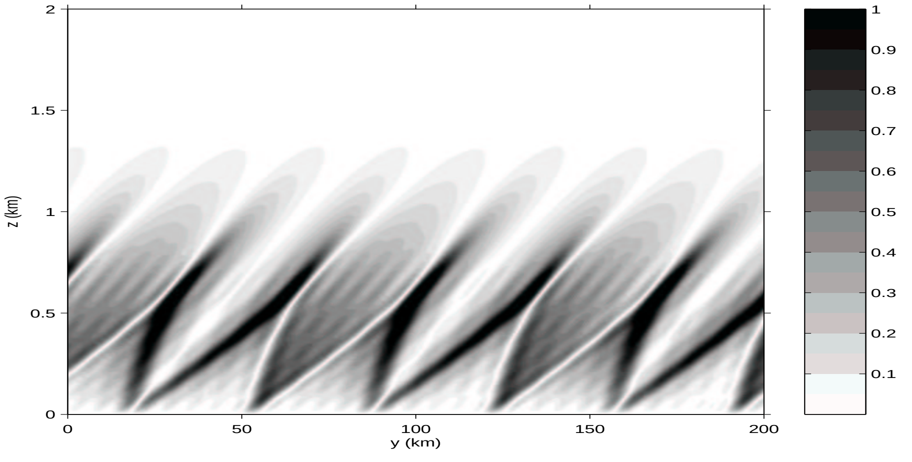

Figure 1. An example of a bottom-trapped sub-inertial wave ($\sigma = 0.99f$), given by a superposition of the £rst £ve modes. The strati£cation is given by the realistic values $N_0 = 5 \times 10^{-4}\mathrm{rad\,s}^{-1}$, gradient $\gamma = 4 \times 10^{-10}\mathrm{rad}^2\,\mathrm{m}^{-1}\mathrm{s}^{-2}$; latitude $\phi = 45^o$ and propagation is in the meridional direction: $\alpha = \pi/2$ (poleward to the right).

velocity and propagate further poleward turning into sub-inertial modes trapped within near bottom and near surface wave-guides. On passing the inertial latitudes the horizontal and vertical scales of waves rapidly decrease and tend to zero at a critical latitude, beyond which there is no wave propagation. The critical latitude is located at a distance a few hundred kilometres from the inertial latitude. The picture resembles wave propagating in a wedge. There is no re¤ection and, thus, inertial waves are absorbed contributing to ocean mixing. Since the main sub-inertial guide is near the bottom, it is expected that the outlined mechanism might be the key one in the so far unexplained mixing in deep ocean.

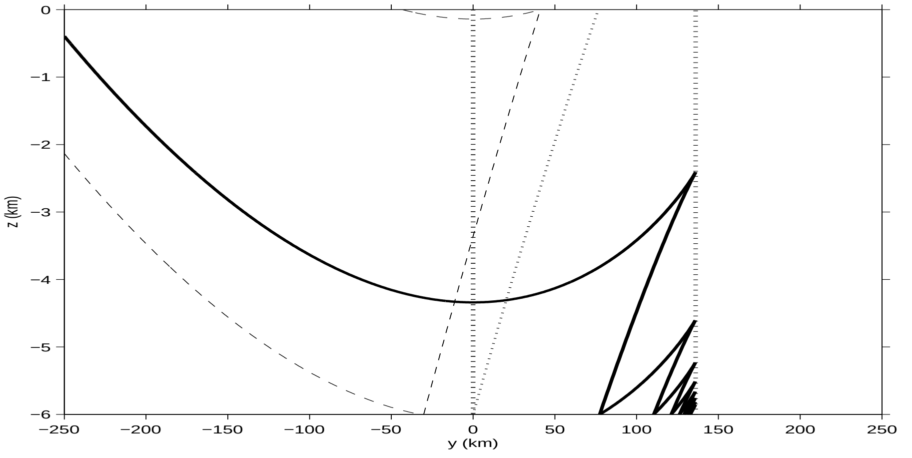

Figure 2. Behaviour of the characteristic curves $\xi_\pm = const$ for the model $N = const$.

The sub-inertial regime is entered via ξ_+ (solid line), which connects to ξ_- as it re¤ects at the critical latitude. The alternation between ξ_+ and ξ_- continues upon each re¤ection at bottom or critical latitude. Inertial and critical latitudes are again indicated by the vertical dotted lines at $y = 0$ and $y = 136$km. A ξ_+-curve that has passed the diagonal dotted line cannot escape anymore from the sub-inertial domain. The dashed line represents an example of an equatorward returning path.The dotted diagonal line representing a $\xi_- = const$ characteristic, marks a borderline: any characteristic $\xi_+ = const$ that re¤ects at the surface to the left of this curve will return equatorward, as is illustrated by the dashed path; all others will get trapped in the lower-right corner of the sub-inertial domain. It thus appears that *most* of the poleward propagating energy will get trapped into this corner.