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LIFETIME PREDICTION WITH A DAMAGE MODEL BASED ON MIXED-MODE MICROCRACK PROPAGATION

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Summary We will first present shortly a framework for brittle damage recently proposed by SCHÜTTE & BRUHNS (2002), where also a more detailed description can be found. The damage evolution equations are based on micro-mechanical considerations. The crack propagation in a unit cell containing a crack is based on a variational principle of an elastic body containing a crack in equilibrium (LE ET.AL., 1999). The thermodynamic equivalence of the micro- and macro-processes of crack growth and damage evolution is used to derive the damage evolution law. We will conclude with numerical examples, which show the possibilities of the proposed model for lifetime prediction.

MACROSCOPIC FRAMEWORK FOR FINITE ELASTODAMAGE

A finite framework for brittle damage is needed to consistently describe the damage process. For the underlying micromechanics see Figure 1.

A multiplicative decomposition of the total deformation can help to describe this process:

$$\mathbf{F} = \mathbf{F}^e\mathbf{F}^d. \tag{1}$$

Here $\mathbf{F}^e$ and $\mathbf{F}^d$ denote the elastic and damage parts of the deformation, respectively. The identification, however, of $\mathbf{F}^e$ and $\mathbf{F}^d$, is quite different from the approach chosen in finite elastoplasticity. $\mathbf{F}^e$ denotes that part of deformation, which leads to elastically recoverable energy and $\mathbf{F}^d$ the one, the energy of which is dissipated.

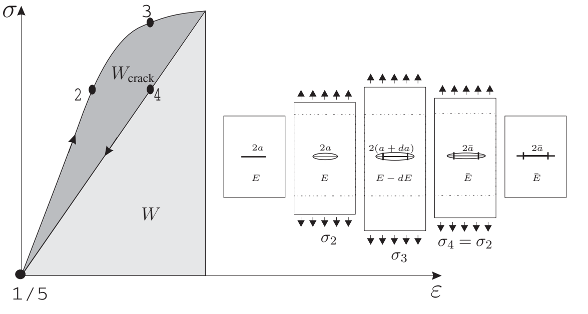

Fig. 1: The stress strain curve resulting from a growing (micro)-crack

Damaged tensor of elastic moduli

We want to show that the elastic properties can be represented in the proposed framework in a straight forward manner by the use of push- and pull operations. The free energy density can be stated as a quadratic form in the intermediate (damaged) configuration. The resulting stress-strain relation for the unloading process is then

$$\bar{\mathbf{s}} = \bar{\rho}\frac{\partial\bar{\psi}}{\partial\bar{\mathbf{e}}^e} = \bar{\mathbb{C}}^e : \bar{\mathbf{e}}; \quad (\mathbb{C}_0^e)^{ABCD} = \lambda\, G^{AB}G^{CD} + \mu\left(G^{AC}G^{BD} + G^{AD}G^{BC}\right)$$

where $\bar{\mathbf{e}}$ is the Almansi-strain of the intermediate configuration, G^{AB} means the components of the reference metric and we postulate isotropic undamaged material in the reference configuration. Damage can be represented by a push-forward with the damage mapping ($\bar{\mathbb{C}}^e = \mathbf{F}^d_{\triangleright}(\mathbb{C}^e_0)$.). The Finger-tensor of the damaged configuration $\bar{\mathbf{b}}^d$, works as an evolving structural tensor. So we get

$$(\bar{\mathbb{C}}^e)^{\alpha\beta\gamma\delta} = \lambda\,(\bar{b}^d)^{\alpha\beta}(\bar{b}^d)^{\gamma\delta} + \mu\left((\bar{b}^d)^{\alpha\gamma}(\bar{b}^d)^{\beta\delta} + (\bar{b}^d)^{\alpha\delta}(\bar{b}^d)^{\beta\gamma}\right), \quad \bar{\mathbf{b}}^d = \mathbf{F}^d_{\triangleright}\left(\mathbf{G}^{-1}\right)$$

which is pointwise orthotropic.

MICROSOPIC FRAMEWORK

2-D crack growth in a unit cell

The micro-mechanical framework is based on the mechanics of a growing crack in a unit cell. The equations for the threshold and direction of crack growth used in the subsequent derivations are based on LE & SCHÜTTE (1998) and LE ET. AL (1999). A crack subject to a mixed mode loading condition can kink at an angle ϕ when it grows. LE & SCHÜTTE (1998) have shown, that the driving force depending on the kinking angle ϕ can be expressed with the help of the SIFs at the kinked crack tip

$$G_\phi = \frac{1-\nu^2}{E}[(K_I^\star)^2 + (K_{II}^\star)^2 + \frac{1}{1-\nu}(K_{III}^\star)^2]. \tag{2}$$

So the crack will grow in the direction which maximizes $G_\phi 4$, if this exceeds certain threshold.

For the problem of an inclined crack in an infinite plate, we know the stress intensity factors prior to crack kinking (IRWIN, 1957) depending on the far-field stresses. The resulting kinking angle from the crack growth law is then

$$\phi_{max} = \mathrm{sgn}(K_{II})[0.71\lambda^3 - 0.0977\sin^2(3.91\lambda) - 13.16\tanh(0.15\lambda)]; \quad \lambda = \frac{|K_{II}|}{K_I + |K_{II}|} \tag{3}$$

which is a curve fitting for the plane part of the solution presented in LE, SCHÜTTE & STUMPF (1999).
After each infinitesimal step of crack propagation, we replace the kinked crack by an equivalent straight crack to take advantage of the analytical equations. Equivalent crack means that it has the same rate of dissipation and amount of additional crack length. The resulting differential equation for the orientation angle is

$$\frac{d\hat{\beta}}{da} = 2\left(G^\star - \hat{J}_1\right) / \frac{\partial \hat{\psi}_{crack}}{\partial \hat{\beta}}, \hat{\psi}_{\text{crack}} = 2\int_0^{\hat{a}} \hat{J}_1 d\hat{a}; \quad \hat{J}_1 = \frac{1-\nu^2}{E}\left(\hat{K}_I^2 + \hat{K}_{II}^2\right), \tag{4}$$

Evolution law for the crack length

What is left up to specify now is an evolution equation for the crack length a. This relation shall be parametrized with the help of the number of cycles N. When the minimum load is such that $G^\star_{\text{min}}$ is smaller than the threshold value for crack propagation $G^\star_{\text{s}}$ the equation can be stated as (compare LEMAITRE & CHABOCHE (1990))

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\tilde{G}^{\star\,\eta/2}_{\text{max}}; \quad \tilde{G}^\star_{\text{max}} = \left\langle \sqrt{G^\star}_{\text{max}} - \sqrt{G^\star_{\text{s}}} \right\rangle^2 \tag{5}$$

MICRO-MACRO-TRANSITION

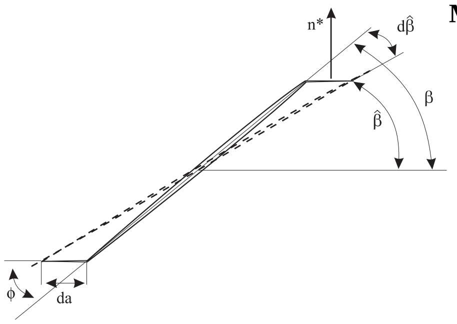

Fig. 2: The kinked and the replacement crack

The transition is done by postulating the equivalence of the macroscopic damage dissipation rate and the microscopic change of the cracks potential

$$\mathcal{D}^d = \bar{\mathbf{k}}^d : \bar{\mathbf{d}}^d = \frac{\dot{\psi}_{\text{crack}}}{A} = \frac{2G^\star \dot{a}}{A}, \tag{6}$$

where A is the area of the unit cell, and t its thickness. This leads to the following damage evolution law

$$\bar{\mathbf{d}}^d = -\frac{\dot{a}}{t}(\mathbf{n}^\star \otimes \mathbf{n}^\star), \tag{7}$$

where $\mathbf{n}^\star$ is the normal vector onto the new developing crack surfaces.

NUMERICAL EXAMPLES

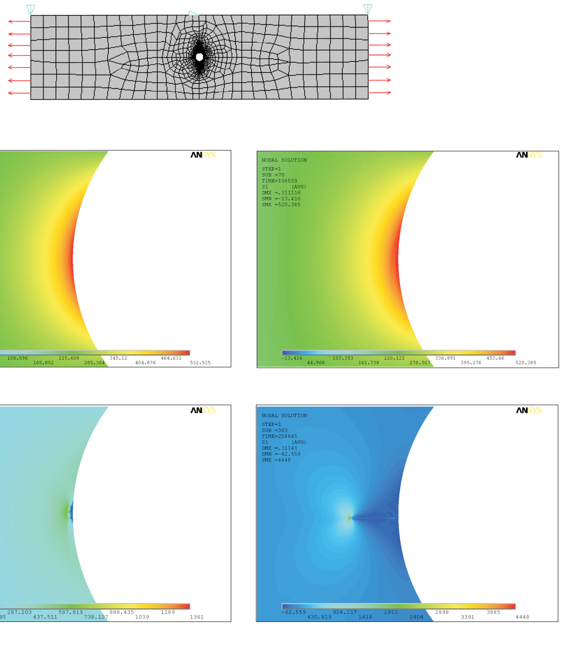

Fig. 4: The specimen and the evolution of stress state over lifetime

The example computed was a strip with a hole under a swelling unidirectional load. The constant maximum stress at the corner of the hole of approx. $533\,$MPa is larger than the chosen fatigue limit of $\sigma_D = 500\,$MPa. The initial crack length of the microcracks is set to $a_0 = 0.001\,$mm. The parameters of the *Paris' law* have been chosen as $C = 0.001$ and $eta = 3.0$. In Figure 2 the stress in loading direction is given for the undamaged state, and three different numbers of cycles: $N = 158559, 243709, 258645$. At $N = 158559$ the stress field is maintained approximately in its undamaged form, but the stress at the corner is slightly reduced ($\approx 520\,$MPa), due to the distributed damage there. At $N = 243709$ one sees clearly that the damage heavily localizes, which results in a sharp crack emanating from the corner of the hole. This macro-crack grows fast and leads to the final failure of the specimen shortly after the stress state shown at $N = 258645$.

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