ACTUATOR AND SENSOR MODELLING FOR LAMINATED PIEZOELECTRIC PLATES

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Summary. We propose an accurate and efficient approach to laminated piezoelectric plates based on a refinement of elastic displacement and electric potential through the plate thickness. Different situations are investigated among them (i) bimorph and (ii) sandwich structures for two kinds of electromechanical loads and are compared to the finite element computations. The prediction of the modal frequencies of vibration of the piezoelectric plate is presented as well.

INTRODUCTION

Due to the successful applications of piezoelectric actuators and sensors, piezoelectric components made of laminated plates have received a particular attention in recent years. The main idea is that certain kinds of structure enable to adapt to or correct for changing operating conditions (geometry, behaviour, etc.) according to electromechanical loads in order to extend the optimum behaviour of the structure in which they are hosted, making them very attractive for innovative technological application. The high performance of piezoelectric composites are used for *multi-purpose devices* or *smart materials* and numerous applications have been proposed, running from aerospace structures (shape control of large space antennas, control of vibrations) to miniature devices due to the possibility of their integration on active electronic circuits (medical apparatus, micro-pump, micro-robots, etc.) [1]

The main objective of the present work attempts to present a consistent and efficient approach to piezoelectric composites made of laminated plate with active piezoelectric layers. Although a number of consistent and accurate approaches to piezoelectric laminated plates have been proposed [2], most of these models are able to accuratly predict the global responses of laminated plates, but they cannot provide excellent estimates of the local responses such as the through-the-thickness variations of the electromechanical quantities and the frequencies of the higher modes of vibration. Here, we propose an alternative approach which combines an *equivalent single-layer representation* for the mechanical displacement with a *layerwise-type approximation* for the electric potential. This approach becomes an interesting feature because multilayered piezoelectric structures are appropriate to accommodate multiple voltage actuator inputs and sensor outputs. Various situations are considered such as bimorph and sandwich structures undergoing two different electromechanical loads (i) surface density of force and (ii) electric potential applied to the piezoelectric layers. In order to show the *quality of predictions* of the present approach, the results are compared to *finite element computations* performed on the 3D problem. In particular, the *global structural response* (deflection, elongation, electric potential or charges, the frequencies of the mode vibrations), as well as the *local response* or the variation of the electromechanical states through the plate thickness are obtained and discussed.

APPROXIMATION OF THE ELECTROMECHANICAL FIELDS

We consider an expansion of the displacements and electric potential as a function of the thickness coordinate as follows

$$\left\{\begin{array}{lcl} u_\alpha & = & U_\alpha - z w_{,\alpha} + f(z)\gamma_\alpha , \qquad \alpha \in \{1,2\} , \\ u_3 & = & w , \\ \phi^{(\ell)} & = & \phi_0^{(\ell)} + z_\ell \phi_1^{(\ell)} + P_\ell(z_\ell)\phi_2^{(\ell)} + g(z)\phi_3^{(\ell)} . \end{array}\right.$$

With $\ell \in \{1, \cdots, N\}$ is the layer number, z_ℓ is the thickness coordinate with respect to the mid-plane of the ℓth layer, U_α is the *middle plane displacement component*, w is the *deflection* and γ_α represents the *shearing function*. The functions of the thickness coordinate are choosen as follows

$$P_\ell(z_\ell) = z_\ell^2 - \left(\frac{h_\ell}{2}\right)^2 , \qquad f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) , \qquad g(z) = \frac{h}{\pi}\cos\left(\frac{\pi z}{h}\right) ,$$

where h is the plate thickness and h_ℓ is the thickness of the ℓth layer and $z_\ell \in [-h_\ell/2, h_\ell/2]$.

However, the continuity of the electric potential as well as the normal component of the electric induction must be satisfied at $z = z_I^{(\ell)}$ ($\ell \in \{1, ..., N-1\}$). The conditions are $\mathcal{A}_\ell = \phi^{(\ell+1)}(x, y, -h_{\ell+1}/2) - \phi^{(\ell)}(x, y, +h_\ell/2) = 0$ and $\mathcal{B}_\ell = D_3^{(\ell+1)}(x, y, -h_{\ell+1}/2) - D_3^{(\ell)}(x, y, +h_\ell/2) = 0$, where the normal component of the electric induction of the ℓth layer is calculated from the constitutive equations. The second condition must be satisfied if no electric potential is applied at $z = z_I^{(\ell)}$, otherwise the condition of continuity is not considered and it is replaced by the jump condition $[\![D_3]\!]_{z=z_I^{(\ell)}} = Q_\ell$ where Q_ℓ is the output surface density of electric charge at the interface $z = z_I^{(\ell)}$. In order to account for the additional conditions, Lagrange multipliers are introduced in the variational formulation [3].

NUMERICAL RESULTS

A collection of benchmark tests is proposed for the piezoelectric bimorph and sandwich structures for plates simply supported under cylindrical bending. Two kinds of electromechanical loads are considered (i) *sensor function* with a surface force density applied to the top face and (ii) *actuator function* with an electric potential applied to the top and bottom faces of the plate and eventually at the layer interfaces [3,4].

In order to illustrate the effectiveness of the proposed approach, we consider, as an example, a laminated plate made of composite material (graphite fibers in an epoxy matrix) for the central layer sandwiched with two piezoelectric PZT layers. The sandwich plate is then subject to a surface density of force normal to the top face. The profile of the induced electric potential at $x = L/2$ within the piezoeletric layers is depicted in Fig.1. For an Applied electric potential the through-the-thickness distribution of the deflection at the plate center is plotted in Fig.2. All these results demonstrate the effectiveness of the present modelling.

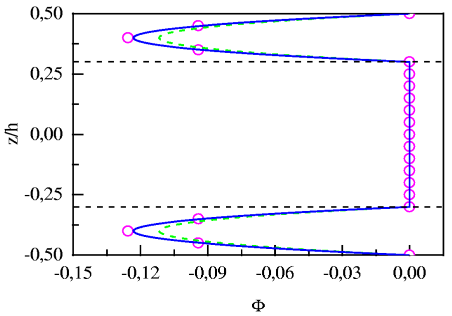

Figure 1. *Sensor function*

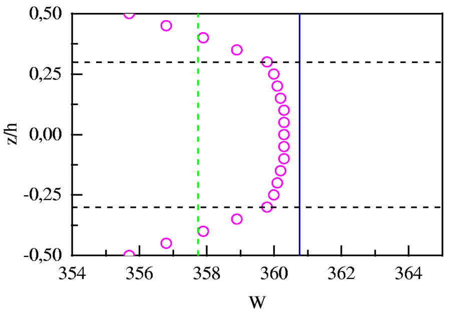

Figure 2. *Actuator function*

We propose the prediction of modal frequencies of piezoelectric plates. Table 1 gives the computational results and comparison to finite element simulations and to simplified model for which no shear correction has been included. The prediction of the modal frequencies for the present approach remains within 5 % for the first seven flexural modes while the simplified model based on the Love-Kirchhoff kinematics plate theory overcomes 20 %. Such an investigation of modal frequencies of vibration is a first step in controlling vibration of elastic structures.

Table 1. *Modal frequencies for piezoelectric sandwich plate (L/h=10).*

Frequencies (Hz) - L/h=10					
MODES	FE	Present Model	Error	Simplified Model	Error
Flex. $n = 1$	29530	29570	0.1 %	29869	1.1 %
Flex. $n = 2$	112624	113295	0.6 %	117537	4.2 %
Flex. $n = 3$	236707	239584	1.2 %	257639	8.1 %
Flex. $n = 4$	388585	396150	1.9 %	442512	12.2 %
Flex. $n = 5$	557423	573241	2.7 %	663571	16 %
Flex. $n = 6$	734569	764062	3.9 %	912474	19.5 %
Flex. $n = 7$	912254	964119	5.4 %	1181874	22.8 %
Axial $n = 1$	324061	324220	0.05 %	324220	0.05 %

CONCLUSIONS

The present study proposes an accurate and efficient approximation for the elastic displacement including a shear function and a layerwise modelling for the electric potential. Such an approach can incorporate the local electromechanical response of the individual layer and becomes a necessity when an electric potential is applied to metallized interfaces between layers. The numerical results and comparisons to FE simulations demonstrate the efficiency of the present model and its capability to accuratly predict the local (field distributions) and global responses.

References

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