

## STRUCTURAL OPTIMIZATION OF COMPOSITE SHELL STRUCTURES USING A DISCRETE CONSTITUTIVE PARAMETERIZATION

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*Summary* In this paper a novel parameterization method for structural optimization of composite laminate shell structures is presented. The method is based on ideas from multi-phase topology optimization where the material stiffness (or density) is computed as a weighted sum of candidate materials. Examples illustrate the potential of the method to solve the problem of proper choice of material, stacking sequence and fiber orientation simultaneously for maximum stiffness or lowest eigenfrequency design.

### INTRODUCTION

The use of fiber reinforced polymers in structural design has gained an ever increasing popularity due to their superior mechanical properties and this work focuses on optimal design of composite laminate shell structures. These structures consist of fiber reinforced polymers stacked in a number of layers and bonded together by a resin, and the design problem is to determine the stacking sequence by proper choice of material and fiber orientation of each layer in order to obtain the desired structural performance. For complicated geometries this is a very challenging design problem that calls for use of sophisticated structural optimization tools.

The major problem when solving such design optimization problems is the non-convexity of the design space, i.e. the risk of ending up with a local optimum solution is high. Several different approaches have been proposed to circumvent this difficulty, and the remedy has typically been to either formulate an optimality criteria method [1, 2], to formulate an equivalent convex problem by introducing lamination parameters [3, 4], or to use gradient based methods, e.g. [5], for example by smoothing the non-convex design space by customizing the optimizer [6]. However, for general shell problems the optimality criteria approaches or the lamination parameter approach have not yet been successfully applied and the customization approach is a highly specialized one, which does not impose convexity.

### THE DISCRETE MATERIAL OPTIMIZATION APPROACH

The design parameterization method suggested in this work is denoted Discrete Material Optimization (DMO), and it is a gradient based technique that can be used for efficient design of general composite laminate shell structures, see [7, 8]. The approach developed is to formulate the optimization problem using a parameterization that allows us to do gradient based optimization on real-life problems while reducing the risk of obtaining a local optimum solution. To this end we will use the mixed materials strategy suggested by Sigmund and co-workers [9, 10] for multi-phase topology optimization, where the total material stiffness is computed as a weighted sum of candidate materials.

In the present context this means that the stiffness of each layer of the composite will be computed from a weighted sum of a finite number of “plausible” constitutive matrices, each representing a given lay-up of the layer. Consequently, the design variables are no longer the fiber angles or layer thicknesses but the scaling factors (or weighting functions) on each constitutive matrix in the weighted sum. For example, we could choose a stiff orthotropic material oriented at three angles  $\theta_1 = 0^\circ$ ,  $\theta_2 = 45^\circ$  and  $\theta_3 = 90^\circ$  and a soft isotropic material, thereby obtaining a problem having four design variables per layer. The objective of the optimization is then to drive the influence of all but one of these constitutive matrices to zero for each ply by driving all but one weight function to zero. As such, the methodology is very similar to that used in topology optimization. This is further emphasized by the fact that penalization is used on the design variables to make intermediate values un-economical. As in topology optimization the parameterization of the DMO formulation is invoked at the finite element level. The element constitutive matrix,  $\mathbf{C}^e$ , in general may be expressed as a sum over the element number of plausible material configurations,  $n^e$ :

$$\mathbf{C}^e = \sum_{i=1}^{n^e} w_i \mathbf{C}_i = w_1 \mathbf{C}_1 + w_2 \mathbf{C}_2 + \dots + w_{n^e} \mathbf{C}_{n^e}, \quad 0 \leq w_i \leq 1 \quad (1)$$

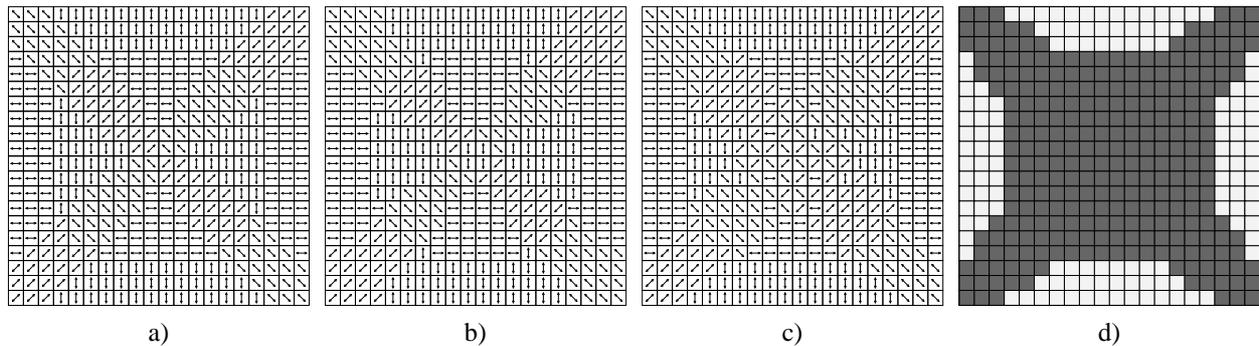
where each “plausible” material is characterized by a constitutive matrix  $\mathbf{C}_i$ . Several new parameterization schemes have been developed, and an example of weighting functions  $w_i$  is given by

$$\mathbf{C}^e = \underbrace{\sum_{i=1}^{n^e} \prod_{j=1}^{n^e} (x_i^e)^p [1 - (x_{j \neq i}^e)^p]}_{w_i} \mathbf{C}_i \quad (2)$$

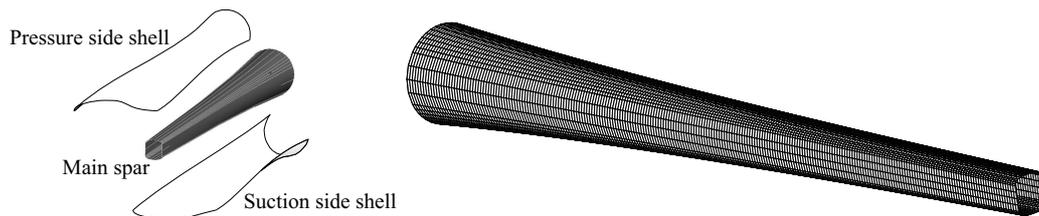
Here  $x_i^e$  represents the element design variables,  $0 \leq x_i^e \leq 1$ , and  $p$  is a penalization power. In case of a mass constraint or eigenfrequency optimization, it is necessary that the sum of the weighting functions equals one. This is not the case for the weighting functions given by Eq. 2, but it can be obtained by normalizing each  $w_i$  by  $\sum_{i=1}^{n^e} w_i$ .

In the examples considered here the design objective is chosen to be a global quantity such as maximum stiffness or lowest eigenfrequency with constraints on the total mass. However, the method is currently being extended to local criteria and geometrically nonlinear structural behavior. Analytical design sensitivity analysis is used and the optimization problem is solved using the Method of Moving Asymptotes [11].

The parameterization is aimed at obtaining practically applicable solutions by choosing the candidate fiber angles to standard integer values (e.g. 0, 45, 90, etc.). The design variables may be associated with each finite element of the model or the number of design variables may be reduced by introducing patches, covering larger areas of the structure. This is a valid approach for practical design problems since laminates are typically made using fiber mats covering larger areas. Several design optimization results are presented in Figs. 1 and 2.



**Figure 1.** Results from material design of clamped square plate. a) Material directions in minimum compliance design of 1-layer plate with uniform pressure and 4 DMO design variables per element associated with an orthotropic material oriented at  $0^\circ$ ,  $\pm 45^\circ$ , and  $90^\circ$ , respectively. b) Material directions in maximum lowest eigenfrequency design of 1-layer plate with the same parameterization as in a). c) Material directions in top layer in minimum compliance design of 4-layer plate with uniform pressure and 5 DMO design variables per layer in each element associated with an orthotropic stiff material oriented at  $0^\circ$ ,  $\pm 45^\circ$ , and  $90^\circ$ , respectively, and a soft isotropic core material, thereby allowing the formation of areas with sandwich structures. The mass constraint allows for 2/3 of the stiff material. After the optimization there is no soft material in the top and bottom layers. d) Distribution of core material in layer 2 and 3 in the example described under c).



**Figure 2.** Model for maximum stiffness design of the load carrying main spar from a wind turbine blade. Optimization is performed using 9600 shell finite elements, and the total number of DMO variables varies from 4312 to 153600 for the test cases studied. Results will be presented at the ICTAM 2004 conference.

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