THE MAGNETOHYDRODYNAMIC COUETTE FLOW IN A PLANE AND SPHERICAL GEOMETRY WITH SINGULAR HARTMANN BOUNDARY LAYERS

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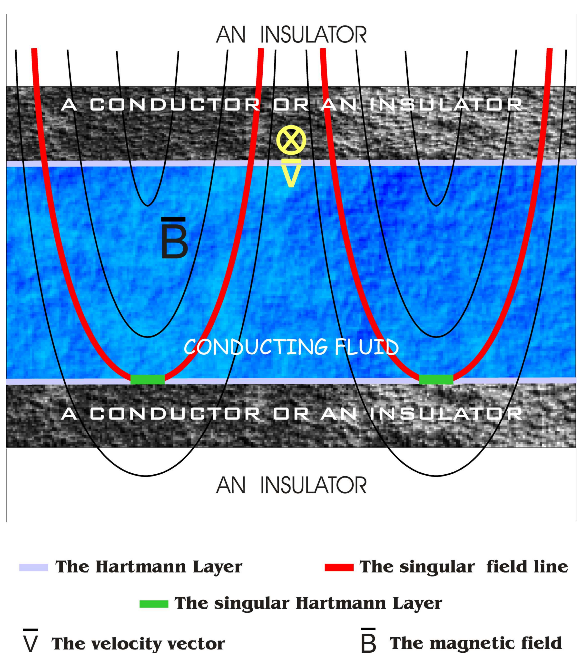
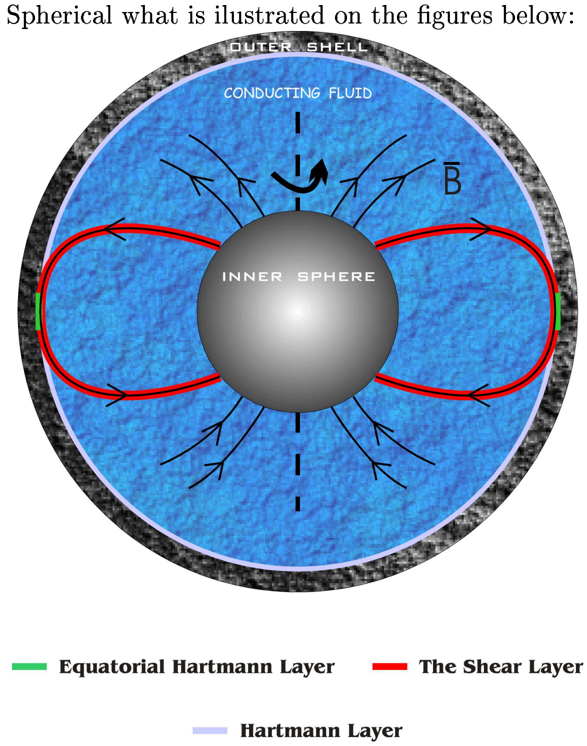
Summary: A magnetohydrodynamic flow between two boundaries one of which is moving with a constant speed is considered here. Two geometries are taken into account: Carthesian and Spherical. The external magnetic field in the system is chosen in such a way that causes the Hartmann boundary layers to become singular at some points. In this configuration the velocity field of the fluid and the infuence of the conductivity of the boundaries on the fluid's movements is studied.

INTRODUCTION AND ASSUMPTIONS

Let's consider a flow of a viscous and conducting fluid between two boundaries which may be either insulating or conducting. One of the boundaries moves *very slowely* at a constant rate what imposes a velocity field in the fluid. The whole system is in *strong* magnetic field chosen in such a way that Hartmann boundary layers which are present in this problem, at some points become singular due to the fact the component of the magnetic field, which is perpendicular to the boundary vanishes. The rest of the space is an insulator.
Two geometries have been taken into account: Carthesian and Spherical what is ilustrated on the figures below:





So the assumptions expressed in terms of the non-dimensional numbers have the form:

$$Re_M = \frac{vL}{\chi} \ll 1 \quad Re = \frac{\rho v L}{\eta} \ll 1 \quad M = \frac{BL}{\sqrt{\mu\eta\chi}} \gg 1$$

where Re and Re_M are the viscous and magnetic Reynolds numbers, v is the velocity of the boundary, L is the thickness of the fluid layer and χ, ρ, η, μ are as follows: the magnetic diffusivity, the density, the viscosity and the magnetic permeability of the fluid.

SOME OF THE RESULTS FOR SPHERICAL FLOW

The solution of the problem stated in the introduction is strongly depended on the character of the boundary - whether it is an insulator, a poor or a good conductor. In this section some of the results for the spherical case (where the external field is assumed to be a dipol field) with slightly conducting outer boundary, meaning that the conductivity of the outer shell is much smaller than the conductivity of the fluid, are presented. Analogical case, but with insulating outer boundary was already resolved by E. Dormy *et al.* (2001), and in this paper I will follow their ideas.

Additional assumptions

In order to make analitical progress some additional assumptions have to be made such as $\epsilon = \frac{\sigma_o}{\sigma} \ll 1$ and $\epsilon M \sim 1$ where σ_o, σ are the electrical conductivities of the outer conductor and the fluid respectively (for simplicity it has been assumed that the conductivity of the inner shere is the same as the conductivity of the fluid).

The summary of results

A boundary layer approximation is used to solve the Navier-Stokes and the induction equations. The boundary layers are of a Hartmann type almost everywhere except for the equatorial region, where the dipol field is tangent to the outer sphere and therefore the boundary layer is singular there. Apart from the boundary layers the velocity and the magnetic field are constant on the dipol field lines therefore the line which is tangent to the outer sphere (red line on the second figure above) has to be treated exceptionally and will be referred to as a critical line.

The influence of the small conductivity of the outer boundary in the Main Flow can be seen only in the magnetic field. The velocity field describes a rigid body rotation with a velocity of the inner sphere due to a strong magnetic interaction between the fluid and the inner conductor.

In the first order of magnitude there is also no influence of the new boundary condition on the solution in the region of the critical dipol field line and in the equatorial Hartmann layer. A non-zero correction to these solutions probably appears in the next order of magnitude.

The most interesting phenomenon in this system are the super-velocities present in the region where the critical dipol field line is tangent to the outer shell. This means that the fluid rotates with the angular velocity greater than the angular velocity of the inner sphere. The solution for these super-velocities is written below:

$$\Omega_S(n) = erf\left(-\frac{n}{\sqrt{8}}\right) - \frac{1}{\sqrt{8\pi}} \int_{-\infty}^{0} \Omega_S(n') \exp\left(-\frac{(n-n')^2}{8}\right) dn' -$$

$$-M^{-\frac{1}{2}} \mathcal{J}_1(1+l, n, M) + M^{-\frac{1}{4}} \mathcal{J}_2(1+l, n, M)$$

where the term $M^{-\frac{1}{4}} \mathcal{J}_2(1+l,n,M)$ is an unknown next order correction outcoming from the fact that the outer boundary is a conductor. Although the precise form of $\mathcal{J}_2(1+l,n,M)$ is not known it can be shown that it is grater than zero, therefore it enlarges the super-rotation, although the factor by which it increases is not evident.

Concluding remarks

This means that the conductivity of the outer boundary has an influence on the magnitude of the super-rotation namely if the conductivity is non-zero the velocities in the equatorial region are larger than in the case of insulating outer boundary.

References:

[1] E. Dormy, D. Jault, A. M. Soward 2001, "A super-rotating shear layer in magnetohydrodynamic spherical Couette flow", *J. Fluid Mech.*