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MEAN ELECTROMOTRICE FORCE FOR A RING OF HELICAL VORTICES

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Summary
We study the dynamo mechanism for a flow made of a ring of stationary helical vortices in an electrically conducting media. The choice of this flow is related to the one obtained in thermal convection in a rotating shell which is also expected in the Earth's outer-core. This choice is also related to a sodium experiment, carried out in Grenoble, based on a spherical Taylor-Couette model. Applying the mean field approach and relying on the second order correlation approximation we derive the mean electromotive force (e.m.f.) produced by such a flow. We find that such a ring of helical vortices may produce, from an azimuthal mean magnetic field, an azimuthal mean e.m.f. leading to the generation of a poloidal magnetic field.

INTRODUCTION

A ring of helical vortices is a common feature of thermal convection in a rotating shell. This fluid motion has been reproduced experimentally and numerically [1] for specific ranges of parameters. The question to know if such a flow is relevant to the flow in the Earth's outercore is not answered yet. However there is some interest to investigate further what kind of dynamo mechanism such a flow could produce. Besides, a sodium experiment in preparation in Grenoble is designed to reproduce such a ring of vortices [2][3]. The device is made of a spherical shell in which there is a rotating innercore. Between the innercore and the spherical shell there is liquid sodium, the whole device being in a rotating frame. The difference of rotation between the innercore and the spherical shell produces a shear instability at the innercore tangential cylinder leading to a ring of vortices. These vortices are helical because of Ekman pumping due to the Ekman layers at the top and bottom and also because of the spherical shape of the top and bottom. We want to know what kind of dynamo mechanism such a flow can produce.

DESCRIPTION OF THE WORK

The mean field approach

We assume a given flow $\boldsymbol{u}$. Instead of calculating directly the magnetic field $\boldsymbol{B}$ by solving the induction equation

$$\partial B/\partial t = \nabla\times(u\times B)+\eta\nabla^2 B \tag{1}$$

where η is the magnetic diffusivity, we use the mean field approach, decomposing $u=\overline{u}+u'$ and $B=\overline{B}+B'$ into mean and fluctuating parts where the mean is defined as the average in the azimuthal direction φ where (s, φ, z) are the cylindrical coordinates. Then our aim is to calculate the mean electromotive force (mean e.m.f.) defined by $\varepsilon=\overline{u'\times B'}$. We can show (see [4]) that it can be expressed in the form:

$$\varepsilon_\kappa(s)=\breve{a}_{\kappa\lambda}(s)\overline{B}_\lambda(s)+\breve{b}_{\kappa\lambda s}(s)\frac{\partial\overline{B}_\lambda(s)}{\partial s}+\cdots \tag{2}$$

where $\breve{a}$ and $\breve{b}$ are pseudo-tensors of rank 2 and 3. Then knowing $\breve{a}$ and $\breve{b}$ the dynamo problem is reduced to solving the mean part of the induction equation:

$$\frac{\partial\overline{B}}{\partial t}=\nabla\times(\overline{u}\times\overline{B})+\nabla\times\varepsilon+\eta\nabla^2\overline{B}\ . \tag{3}$$

Assumptions

Let us write the fluctuating part of the induction equation:

$$\frac{\partial B'}{\partial t}=\nabla\times(u'\times\overline{B})+\nabla\times(\overline{u}\times B')+\nabla\times(u'\times B')-\nabla\times(\overline{u'\times B'})+\eta\nabla^2 B' \tag{4}$$

Then we consider the low conductivity limit (see [4]), leading to neglecting the term at the left hand side of (4). We also consider that the second order approximation (see [4]) is valid (leading to neglecting the 3rd and 4th terms of the right hand side of (4)). Finally, we consider that the mean e.m.f. does not depend on $\overline{u}$ (see [4]), assuming that it is sufficient to consider $\overline{u}$ in (3) only.
Finally we come out with the following equation to solve:

$$\eta\nabla^2 B'=-\nabla\times(u'\times\overline{B})\,. \tag{5}$$

The velocity field

The flow is considered to be steady, z-independent and harmonic in φ. It is non-zero for $1-\delta\le s/l_0\le 1+\delta$ where l_0 is a typical length scale defined by the radius of the ring of vortices. The velocity is defined by $u=(u_s(s)\sin m\varphi, u_\varphi(s)\cos m\varphi, u_z(s)\cos m\varphi)$ and a typical example is given in figure 1.

Tensor of second rank

Solving (5) and replacing *B'* in $\varepsilon = \overline{u' \times B'}$, we find that $\breve{a}_{\kappa\lambda} = \frac{\eta}{l_0} R_m^H \begin{pmatrix} R_m^Z \tilde{a}_{ss} & 0 & 0 \\ 0 & R_m^Z \tilde{a}_{\varphi\varphi} & 0 \\ 0 & R_m^H \tilde{a}_{z\varphi} & 0 \end{pmatrix}$ with $R_m^H = \frac{u_0^H l_0}{\eta}$ and $R_m^Z = \frac{u_0^Z l_0}{\eta}$, u_0^H and u_0^Z being typical horizontal and vertical velocities, and with:

$$\tilde{a}_{ss} = -\frac{1}{2}\int_0^\infty \left(\frac{\partial h_m(s,s')}{\partial s'} u_\varphi(s) u_z(s') + \frac{\partial h_m(s,s')}{\partial s'} u_z(s) u_\varphi(s') \right) s' ds'$$

$$\tilde{a}_{\varphi\varphi} = \frac{m}{2}\int_0^\infty \left(\frac{h_m(s,s')}{s} u_z(s) u_s(s') + \frac{h_m(s,s')}{s'} u_s(s) u_z(s') \right) s' ds'$$

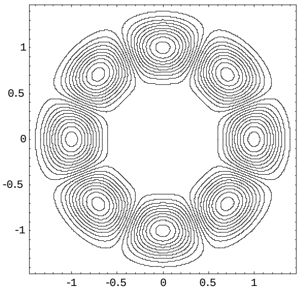
$$\tilde{a}_{z\varphi} = -\frac{1}{2}\int_0^\infty \left(\frac{\partial h_m(s,s')}{\partial s} u_s(s) u_s(s') + m \frac{h_m(s,s')}{s} u_\varphi(s) u_s(s') \right) s' ds'$$

and where $h_m(s,s') = \frac{1}{2m}\left(\frac{s'}{s}\right)^m$ for $s' \le s$ and $h_m(s,s') = \frac{1}{2m}\left(\frac{s}{s'}\right)^m$ for $s \le s'$.

The tensor of third rank has also been calculated but cannot be included in this short summary.

One example

As an example, we consider the velocity profile defined by $u_z / u_0^Z = \frac{15\pi}{16}(1-\xi^2)^2 \cos m\varphi$ and $(u_s, u_\varphi, 0)/u_0^H = -e_z \times \nabla\psi$ with $\psi = \delta(1-\xi^2)^3 \cos m\varphi$ and $\xi = \frac{s-1}{\delta}$. The *s*-profiles obtained for the three coefficients $\tilde{a}$ are given in figure 2 in the case where $\delta m = \pi/2$, corresponding to helical vortices with similar length scales in the *s* and φ directions.



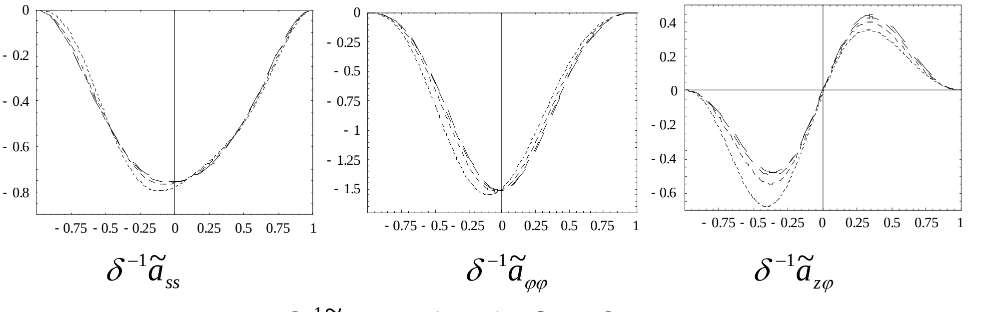


Fig.1. Isolines of u_z for *m*=4. Fig.2. The functions $\delta^{-1}\tilde{a}$ versus $(s-1)/\delta$ for $\delta m = \pi/2$ and *m*=1, 2, 4, 8, 16

CONCLUSIONS AND FURTHER WORK

The coefficient $\breve{a}_{\varphi\varphi}$ being not zero and even dominant compared to the others, we can conclude that from an azimuthal mean magnetic field an azimuthal mean e.m.f. is generated. Then this e.m.f. can generate a poloidal magnetic field. The generation of a azimuthal field from a poloidal field by some differential rotation $\overline{u}$ could then close the loop. We are now considering the case where the flow is z-dependent. In the future we shall examine the influence of $\overline{u}$ onto the mean e.m.f.

References

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