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THERMOCHEMICAL CONVECTION IN TWO SUPERIMPOSED MISCIBLE VISCOUS FLUIDS

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Summary Marginal stability analysis and laboratory experiments have been performed to investigate thermal convection in two superimposed layers of miscible fluids. Two different regimes are observed: a whole-layer regime, where the interface deforms and convecting patterns develop over the whole depth of the system, and a stratified regime, where convection develops above and below a stable interface. Various behaviours are described and scaling laws determined.

INTRODUCTION

The interest in two-layer thermal convection has been largely inspired by natural problems, in particular the dynamics of the Earth's mantle [1]; besides, it has also been a theoretical challenge, because of the possibility of Hopf bifurcation and time-dependence at marginal stability [2]. The simple fact of adding a second layer considerably complicates the problem of thermal convection and opens up a very large parameter space that has not yet been fully explored.

In our study, we consider two superimposed layers of miscible fluids, with different densities, viscosities and depths, which are heated from below and cooled from above. All other properties of the fluids are equal. Besides, only the case where the initial density stratification is stable is studied; this configuration can however be reversed by thermal effects.

MARGINAL STABILITY ANALYSIS

Equations of motion correspond to Navier-Stokes equations in each fluid, coupled at the deformable interface. Apart from the Prandtl number (taken as infinite), four dimensionless numbers are necessary to fully describe the system: the viscosity ratio γ and the layer depth ratio a characterize the differences between the two layers, the Rayleigh number Ra measures the strength of convection, and the buoyancy number B, ratio of stabilizing chemical to destabilizing thermal density anomalies, determines the stability of the whole system and the ability of the interface to deform.

Linear stability analysis [3] points out the possible occurrence of two different regimes depending on B (see figure 1): i) when B is larger than a critical value B_c depending both on viscosity and layer depth ratios, a stratified regime takes place, with convecting patterns developing above and below a stable interface; ii) when B is lower than the critical value, a whole-layer regime takes place, with a deformed interface and convecting patterns developing over the whole depth of the system.

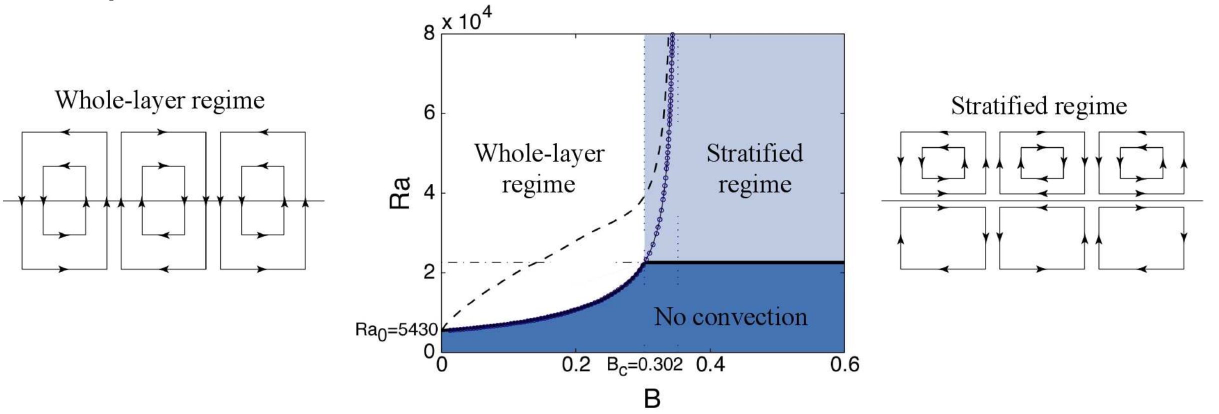

Figure 1: marginal stability analysis for γ=6.7 and a=0.5

EXPERIMENTAL STUDY

Using laboratory experiments [4-5], we have explored large ranges of buoyancy and Rayleigh numbers in order to characterize all possible behaviors ($0.048<B<4.4$ and $32<Ra<6.1\times10^8$). Close to marginal stability, the early scales of convection are well predicted by the linear analysis. At large Rayleigh number Ra, the situation is complicated by the superimposition of various types of convective features: only looking at one of the two fluids, the destabilization of its outer thermal boundary layer possibly leads to the formation of small-scale plumes as in classical Rayleigh-Bénard convection, thus referred as 'purely thermal'; but purely thermal features from hot and cold plates also interact at the interface, where they induce a large-scale thermochemical regime, either with a stable interface (even if partly deformed), thus corresponding to the stratified regime, or with a fully destabilized interface, thus corresponding to the whole-layer regime. When $1/5<\gamma<5$ typically, the whole-layer regime takes the form of large convective features developing through the whole depth of the tank. The interface is distorted in all directions, and the two-layer initial system is never reconstructed: overturning and immediate mixing operates. When $\gamma>5$ or $\gamma<1/5$ typically, the whole-layer regime gives rise to large-scale oscillations: the two fluids conserve their own identity, and the initial two-layer system is periodically reconstructed. The number of observed pulsations rapidly increases with γ. Two mechanisms of initial system reconstruction are possible (figure 2):

- vertical oscillations: starting from a stratified system, the lower fluid is progressively heated and becomes lighter, whereas the upper fluid is cooled and becomes heavier. Once the chemical density anomaly is cancelled by thermal effects, the interface deforms in large domes that rapidly propagate until they reach the opposite boundary: fluid 1 near the cold plate becomes heavier whereas fluid 2 near the hot plate becomes lighter. The initial stratification finally reappears and the system goes back to its initial configuration. A new oscillation can begin. Stirring between the two layers of course slowly works by advection, but more than 10 successive pulsations have been observed.
- initial configuration reversals: the whole invading layer is progressively emptied, until the initial configuration is totally reversed, with fluid 1 lying above fluid 2. Then, fluid 1 cools down, fluid 2 heats up, and the system finally goes back to initial state. In this case, stirring also works by advection, but several successive reversals can be observed.

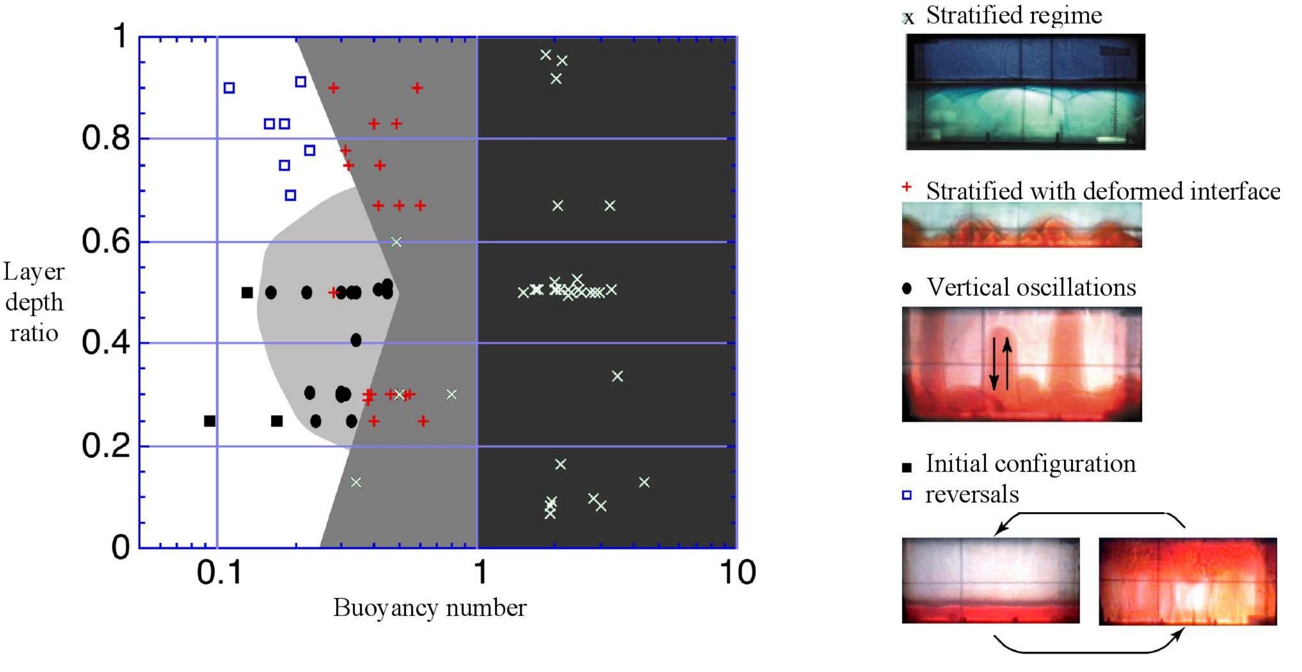

Figure 2: observed regimes depending on the buoyancy number and layer depth ratio.

Initial configuration reversals take place when the chemical stratification is low compared to the thermal buoyancy (B<0.2 typically and/or large Ra), but also when the invading layer is small and thus rapidly emptied (a<0.3 or a>0.7). Vertical oscillations take place otherwise. In all cases, the characteristics of domes (speed, size, period) are mainly controlled by the most viscous layer, which slows down motions over the whole depth of the tank.

CONCLUSION AND IMPLICATIONS FOR EARTH'S MANTLE

All described regimes are transient and the system systematically evolves towards one-fluid Rayleigh-Bénard convection because of stirring. However, the two isolated fluids can persist for very long time compared to the characteristic time-scale of thermal convection. Of particular interest for Earth's mantle are the vertical oscillations [6]: according to the scaling laws derived from the experiments, such a mechanism could indeed provide a simple and single physical explanation for the long-term periodicity (i.e. periods of hundreds millions years) observed in volcanic activity at the surface and magnetic activity in the core.

References

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