

## PLASTIC DEFORMATION BY IMPACTS IN MULTIBODY SYSTEMS

Robert Seifried, Werner Schiehlen

*Institute B of Mechanics, University of Stuttgart, 70569 Stuttgart, Germany*

*Summary* In multibody system dynamics the coefficient of restitution is used to describe the energy loss during impact. A numerical method is presented to evaluate the coefficient of restitution considering plastic deformation in the contact region and wave propagation in the bodies. The longitudinal impact of a steel sphere on different aluminum rods is used as testing example for the presented approach and the results are compared with measurements.

### INTRODUCTION

In machine dynamics, the multibody system approach proves to be most efficient for the dynamical analysis of the overall motion. Collisions in multibody systems might result in impacts between the system's bodies. During impact kinetic energy is lost, described macro-mechanically by the coefficient of restitution [1]. The loss of kinetic energy is due to local plastic deformation resulting from high forces in the contact region and the initiation of elastic waves in the bodies. After separation of the bodies the waves die away due to material damping. For impacts of compact bodies, e.g. the impact of two spheres, the main source of kinetic energy loss is plastic deformation of the contact region, whereas for impacts involving slender bodies, such as rods, beams, plates or shells a significant amount of the initial kinetic energy is transformed into waves. The coefficient of restitution cannot be computed within the multibody system approach, but it has to be measured experimentally, estimated from experience or evaluated numerically by additional simulations on a fast time scale, resulting in a multi-time-scale simulation [2], respectively.

### SIMULATION ON THE FAST TIME SCALE

For an accurate numerical evaluation of the coefficient of restitution the simulation models on the fast time scale must be capable to represent the high-frequency phenomena of wave propagation as well as the local plastic deformation in the contact region. For impacts of simple shaped bodies, such as the impact of a sphere on a rod, the equation of motion for elastodynamics can be introduced and solved by D'Alembert's approach for wave propagation combined with the Hertzian contact law [3]. However, the Hertzian contact law is restricted to elastic contact with spherical contact geometry. For the impact of more complex shaped bodies and more complex contact conditions the Finite-Element-Method can be used [4]. The contact regions of the discretized bodies are connected by contact elements, which are often based on a penalty formulation. Material nonlinearities, such as plasticity can be included in the simulation, too. However, due to the high frequency phenomena resulting from the collision a globally small element length and a small integration time step size are required to catch the wave propagation correctly. This results in a high computation time. Using a modal approach for the colliding bodies, the elastodynamic behavior can be simulated more efficiently. Using the Finite-Element-Method for the computation of the mode shapes, complex shaped bodies can be simulated, too. The contact can be described by a compliance law such as the Hertzian contact law. For the inclusion of plasticity in the contact region a hysteretic force-indentation law might be used, see e.g. [5], however, these laws require parameters which cannot be computed from the yield stress and, thus, have to be measured experimentally.

In this paper a combination of the modal approach, describing the global elastodynamic behavior of the bodies and a Finite-Element simulation of the contact region is presented, using the longitudinal impact of a steel sphere on an aluminum rod as testing example. The sphere is modeled as a rigid body whereas the rod is represented by the modal approach. The rods eigenfrequencies and mode shapes are computed by a FE-model of the rod. Using the modal approach the displacement of a point of the discretized rod is given by the sum of the rod's rigid body motion  $x_1(t)$  and the first  $n$  eigenmodes given by the previously computed mode shapes  $\phi$  and the corresponding modal coordinates  $q_i$ ,

$$u_p(t) = x_1(t) + \sum_{i=1}^n \phi_{ip} q_i(t), \quad p = 1, 2, \dots$$

where the index  $p$  identifies the nodes of the rod's FE model. For the modeling of impacts frequencies up to 100kHz are taken into account. The investigated rod is suspended like a pendulum, thus the equations of motion of the rod reads as

$$m \ddot{x}_1 + m \frac{g}{l} x_1 = F(t)$$

$$\ddot{q}_i + \omega_i^2 q_i = f_i, \quad i = 1(1)n,$$

where  $m$  is the rod's mass,  $g$  the gravity,  $l$  the length of the pendulum,  $F$  the contact force and  $\omega_i$  the angular frequency of the  $i^{\text{th}}$  eigenmode. The modal contact force is given by  $f_i = F \phi_{ic}$  where  $p=c$  denotes the node at the point of the contact of the modal model. The contact force  $F$  is computed in parallel by a FE-simulation of the contact region using ANSYS including plastic material properties. The modal model and the FE-model of the contact region are connected in each integration time step by exchanging displacements of the boundary of the contact region and the contact force as shown schematically in Figure 1.

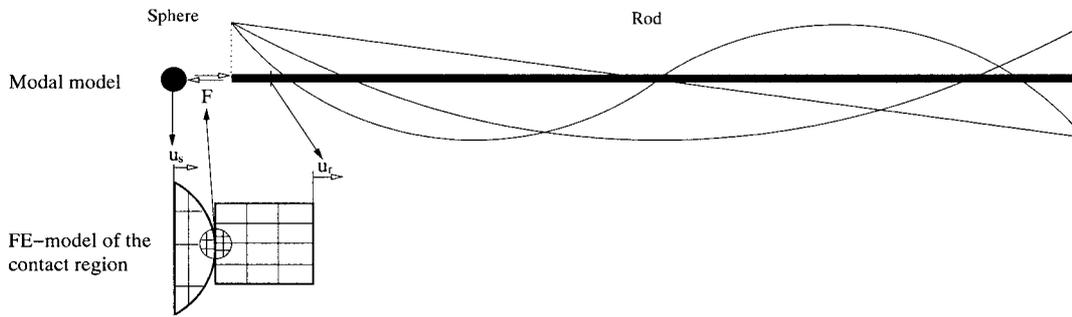


Figure 1: Schematic presentation of the combined simulation procedure

Comparisons of the combined simulation procedure with plain FE-simulations show good consistency, however, especially for larger models the computation time is reduced significantly. By removing the FE-model of the contact region after impact the post impact behavior, i.e. the vibration of the solid bodies, can be simulated even more efficiently.

### COEFFICIENT OF RESTITUTION

From the impact force computed on the fast time scale the coefficient is determined [3] and then fed back to the multibody system simulation. In Figure 2 the computed coefficients of restitution for the impact of a steel sphere of radius 15mm on two different aluminum rods are presented for a velocity range from 0.025m/s to 1m/s. Both rods have a radius of 10mm and a length of 1m, however rod 1 has a yield stress of 575Mpa whereas the yield stress of rod 2 is 204Mpa. In both plots the results of purely elastic and elastic-plastic simulations are presented. Rod 1 shows for a wide range elastic material behavior, i.e. the energy loss are mostly due to wave propagation; rod 2 shows already for very low velocities plastic deformations. The presented combined model is also modified to compute multiple impacts of the same bodies including the deformation history of the contact region. It is shown that in a series of successive impacts on a body the contact region gets plastically deformed by the first few impacts until a configuration is reached where no additional plastic deformation occurs, in these examples after maximal 10 impacts. In this phase the remaining energy loss is due completely to wave propagation. Therefore, the coefficient of restitution changes with the number of

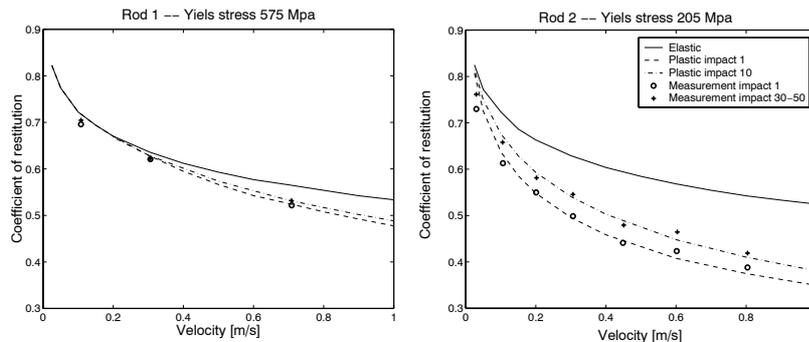


Figure 2: Computed and measured coefficients of restitution

impacts until a stationary value is reached. In Figure 2 measured values using Laser-Doppler-Vibrometers [3] are indicated, too. They show good agreement with the plastic simulation results. Also the increase of the coefficient with the number of impacts can be observed. Due to slight variations of the contact point during the experiments the stationary value for coefficient of restitution is reached after a larger number of impacts than predicted in the simulations.

### CONCLUSIONS

A simulation technique for the evaluation of the coefficient of restitution is presented using a model combining the modal approach for the elastodynamic behavior of the bodies and a FE-model including plastic material behavior for the contact region. Also multiple impacts of the same bodies including the deformation history are analyzed. Longitudinal impacts of a sphere on rods are used as testing example and compared with experiments. It is shown that a substantial amount of kinetic energy is lost into wave propagation and in plastic deformation, depending on the material's yield stress. With the number of impacts the coefficient of restitution is increasing due to hardening phenomena.

### References

- [1] Pfeiffer F., Glocker C.: Multibody Dynamics with Unilateral Contacts. Wiley, New York, 1996.
- [2] Schiehlen W., Seifried R.: Multiscale impact models: Interaction between multibody dynamics and wave propagation. In: *Proceeding of the IUTAM Symposium on Nonlinear Stochastic Dynamics*. Kluwer, Dordrecht, 353-362, 2003.
- [3] Hu B., Eberhard P., Schiehlen, W.: Comparison of analytical and experimental results for longitudinal impacts on elastic rods. *Journal of Vibration and Control*, 9, 157-174, 2003.
- [4] Seifried R., Hu B., Eberhard P.: Numerical and experimental investigation of radial impacts on a half-circular plate. *Multibody Systems Dynamics*, 9, 265-281, 2003.
- [5] Lankarani H.M., Nikravesh P.E.: Continuous Force Models for impact analysis in multibody systems. *Nonlinear Dynamics*, 5, 193-207, 1994.