OSCILLATORY MOTION OF FREELY-MOVING LIGHT BODIES: FROM CYLINDERS TO DISKS

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Summary We report on an experimental investigation of the oscillatory motions of flat cylinders rising in a low-viscosity fluid otherwise at rest. Original results are presented concerning the vertical velocity and the horizontal oscillations of the body velocity and orientation (amplitude, frequency and phase difference), underlining the crucial effect of the body aspect-ratio on the complex coupling between body rotation and translation.

In many situations, free falling or rising particles at moderate-to-large Reynolds numbers exhibit oscillatory motions: spiral, zigzag or tumbling. Our general concern is to determine the causes of these oscillations for particles moving under the effect of buoyancy in a fluid otherwise at rest. Consider first a fixed sphere of radius r in a uniform flow of velocity V of a fluid of viscosity ν. Beyond a certain Reynolds number ($Re = 2rV/\nu$), the wake becomes unstable and unsteady drag and lift forces act on the body. For a freely-moving buoyant sphere [1], these forces represent a first possible cause of path oscillations. When the sphere moves freely, its velocity V is *a priori* unknown and the Reynolds number is replaced by the Archimedes number, $Ar = r\sqrt{\frac{\Delta\rho}{\rho_l}gr}/\nu$ (where g is the acceleration of gravity and $\Delta\rho$ the density difference). As the amplitude of the oscillations is significant only if the body is light enough, the solid-to-fluid density ratio ρ_s/ρ_l is an important parameter. For a non-spherical body, the anisotropy of the added-inertia tensors can also produce path oscillations because it couples the equations for translation and rotation. Moreover, the viscous drag also depends on the direction of the velocity with respect to the body orientation. Therefore, the body shape is also of major importance.

We carried out an experimental study on the rise of light flat cylinders (diameter d, thickness h and equivalent radius $R_{eq} = \left(\frac{3}{16}hd^2\right)^{\frac{1}{3}}$) in a 1.70 m high tank of salted water. Since our objective is to investigate the range of moderate Archimedes numbers, we used very small density differences ($10^{-3} \leq \Delta\rho/\rho_l \leq 10^{-2}$) determined with an accuracy of 5×10^{-4} kg/dm^3. Consequently, ρ_l/ρ_s is very close to unity. Experiments were performed for various aspect ratios d/h ranging from 1.5 to 10 and Ar between 50 and 100 ($150 \leq Re \leq 300$). The vertical mean velocity, V_z, ranges between 10 to 35 mm/s and the amplitude of horizontal body-centre displacements lies between 0.15 and 3 mm, which corresponds to velocities between 0.5 and 5 mm/s. The rise of the body is followed by two moving cameras; its location and orientation are then determined by image processing with an accuracy of ± 0.15 mm (see figure 1). We observe that in all cases orientation and horizontal velocity experience harmonic oscillations at the same frequency ω, the vertical velocity oscillating at 2ω. The frequency ($0.1 \leq \omega \leq 0.35$ Hz) is an increasing (resp. decreasing) function of Ar (resp. d/h). However, the Strouhal number, $St = R_{eq}\,\omega/V_z$, is approximately independent of Ar and remains close to 0.06 for $d/h \geq 4$.

We consider now the amplitudes of the oscillations. From a top view, the trajectory is an ellipse, the principal axes of which we denote by X and Y. The displacements in the X- and Y-directions are described by sine functions (with amplitudes A_X and A_Y) that are $\pi/2$ out of phase. The ratio A_Y/A_X is observed to vary from 0 (plane zigzag trajectories) to 1 (circular helix). For Ar larger than 70, A_Y/A_X first increases from 0 to 0.35 when d/h is increased from 1 to 4 and then decreases towards 0 when d/h is further increased. For lower values of Ar, A_Y/A_X seems to be an increasing function of d/h over the whole range of aspect ratio investigated but the conclusion is uncertain for d/h larger than 8 because the amplitude of oscillations becomes very small.

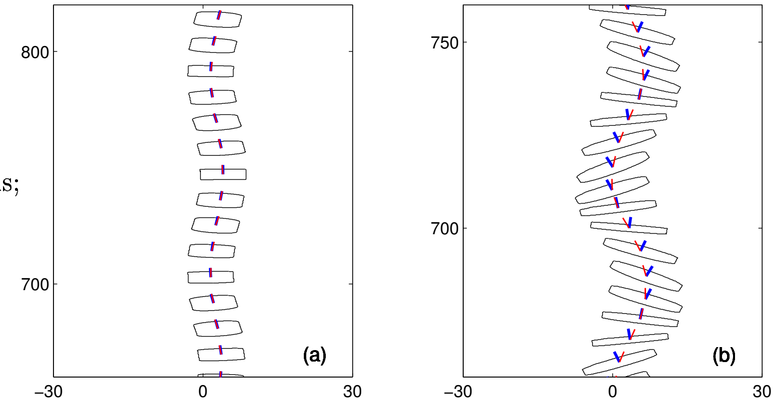

Figure 1. Typical body motions ($Ar = 85$). Thick blue line: body axis; Thin red line: velocity direction. (a) $d/h = 2$; (b) $d/h = 10$.

The amplitude of the horizontal displacement is represented on figure 2(a) by the maximum angle, θ_{max}, made by the body-center velocity and the vertical direction. The amplitude θ_{max} increases with Ar but exhibits two different behaviors depending on d/h being larger or smaller than 4. Here all the aspect ratios investigated correspond to oblate bodies, the axis of which oscillates about the vertical direction. Note that the limit $d/h \to 0$ corresponds to a strongly different behavior since a stable prolate body (as a rod) moves with its axis perpendicular to its velocity. It is thus not surprising to observe a monotonic evolution towards the disk limit ($d/h \to \infty$) only beyond $d/h = 4$; after this value θ_{max} increases strongly. The same behavior is observed for vertical velocity fluctuations which are negligible for d/h less than 4 and then increase regularly with d/h. However, the amplitude of the vertical oscillations is smaller (at least 5 times) than the horizontal one which itself is smaller (at least 3 times) than the mean rise velocity V_z. It is thus relevant to examine the drag coefficient C_d based on the mean velocity. At a given Ar, the two regimes are again observed: C_d first increases and keeps a constant value beyond $d/h = 4$. More interesting is the fact that for given d/h, C_d is an increasing function of Ar and consequently of Re. This seems in contradiction with results concerning fixed bodies. On the other hand, experiments [2] and numerical simulations [3] concerning rising bubbles show that the drag coefficient is an increasing function of the amplitudes of path oscillations. This is also observed here.

Figure 2(a) also shows the evolution of the maximum angle, α_{max}, made by the body axis and the vertical direction. The two distinct regions separated by $d/h = 4$ are again observed but at variance with velocity oscillations, α_{max} is approximately constant for $d/h \geq 4$. In other words, we observe that when the body becomes thinner the amplitude of the horizontal displacement increases whereas its maximal inclination remains almost constant. An important feature of figure 2(a) is that the instantaneous velocity is not parallel to the body axis. The drift angle between the velocity and the body axis is crucial for a non-isotropic body since it couples translation and rotation. However, the difference between α_{max} and θ_{max} is not sufficient to characterize this drift angle, since the phase difference between the harmonic oscillations of the velocity and those of the body orientation depends on d/h. Indeed, figure 1(a) shows that for a thick body ($d/h = 2$) velocity and orientation are almost in phase whereas figure 1(b) shows that for the thinnest body ($d/h = 10$), they are about $\pi/2$ out-of-phase. In figure 2(b) one sees that the phase difference varies continuously with d/h between these two limits and is almost independent of Ar.

The detailed description of the oscillatory motion of rising flat cylinders carried out in this work for a wide range of aspect ratio and Archimedes number underlines the complexity and diversity of the dynamics of rising bodies. Concerning the mean rise velocity, it appears that the drag coefficient is not a simple function of the mean Reynolds number, as for a fixed body, but is strongly influenced by the oscillatory motion. When d/h is increased the amplitude of body orientation oscillations reaches a constant value while that of velocity oscillations continues to increase. Moreover, the phase difference between the body velocity and orientation oscillations evolves from about 0 (bubble-like behavior) to a value close to 90^o (disk-like behavior) when the aspect ratio is increased. However, the orientation and the velocity oscillate at the same frequency, characterised by a Strouhal number rather independent of both Ar and d/h for $d/h \geq 4$ and have amplitudes that increase with Ar. This suggests that the wake instability mechanism, which is related to the production of vorticity at the body surface and generates the oscillations, is rather independent of the body shape (for $d/h \geq 4$) and drives the body velocity and rotation to which it imposes its frequency. The body velocity and rotation respond to this forcing with their respective dynamics, which strongly depend on the body aspect ratio and determine their own amplitude and phase. To check this interpretation, our future work will focus on the determination of the different hydrodynamical forces and couples that act on the body, the present experimental techniques being sufficiently accurate to determine the linear and angular accelerations of the body.

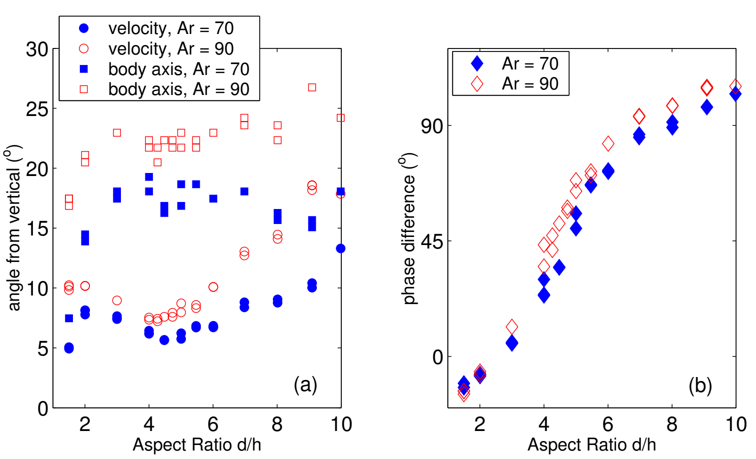

Figure 2. Comparison between body axis and horizontal velocity oscillations:
(a) amplitudes measured by maximum angles from the vertical direction;
(b) temporal phase difference.

References

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