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WAVE CAPTURE AND WAVE–VORTEX DUALITY

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Summary
New and unexpected results are presented regarding the nonlinear interactions between a small-scale wavepacket and a large-scale mean flow, with an eye towards internal wave dynamics in the atmosphere. These are to do with an unusual wave breaking scenario termed 'wave capture', which differs significantly from the standard wave breaking scenarios due to critical layers or mean density decay. We focus on the very peculiar wave-mean interaction scenario that accompanies wave capture.
We present examples of these interactions in 2d shallow water and in the 3d Boussinesq system. Specifically, we point out an analogy between slow wavepackets and vortex dipoles that shows a peculiar and apparently so-far unrecognized 'wave-vortex duality', which throws a new light on the relation between wave dissipation and mean-flow forcing.

Wave capture and wave–mean interaction

This is on-going work in wave–mean interaction theory based on [2]; the new results are reported in full in [3]. The basic idea [1] is to exploit the partial analogy between the evolution of the wavenumber vector $\boldsymbol{k}$ in ray theory with a non-uniform mean flow $\boldsymbol{U}$ and the evolution of the gradient of a passive tracer advected by the same flow:

$$\left(\frac{\partial}{\partial t} + ([\boldsymbol{U} + \hat{\boldsymbol{u}}_g] \cdot \boldsymbol{\nabla})\right) \boldsymbol{k} = -\boldsymbol{\nabla}\boldsymbol{U} \cdot \boldsymbol{k} \quad \text{and} \tag{1}$$

$$\left(\frac{\partial}{\partial t} + (\boldsymbol{U} \cdot \boldsymbol{\nabla})\right) \phi = 0 \Rightarrow \left(\frac{\partial}{\partial t} + (\boldsymbol{U} \cdot \boldsymbol{\nabla})\right) (\boldsymbol{\nabla}\phi) = -\boldsymbol{\nabla}\boldsymbol{U} \cdot (\boldsymbol{\nabla}\phi). \tag{2}$$

The only difference is the intrinsic group velocity $\hat{\boldsymbol{u}}_g$. Therefore, for slow wavepackets such that $\hat{\boldsymbol{u}}_g \ll \boldsymbol{U}$ the analogy is very good. This implies the readily verified potential for exponentially fast growth in $\boldsymbol{k}$ and concomitant growth in wave amplitude, which must lead to rapid wave breaking [1]. We call this basic effect "wave capture" here, to emphasize that the wavepacket gets captured by (and subsequently travels with) the mean flow. We stress that capture can occur without the need for any critical layer in the classical sense, and without any density decay with altitude.
This capture has profound implications for pseudomomentum budget and wave–mean interaction, and these will be discussed. A detailed example is illustrated in figure 1, which is taken from [3].

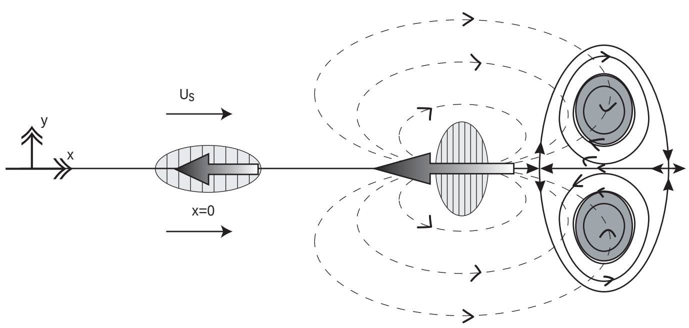

Figure 1. Horizontal isentropic view of a mountain wavepacket that was generated below by a surface wind U_s over topography and which is now undergoing capture due to the blocking dipole on the right. The wavepacket drifts towards the dipole stagnation point whilst being strained by the mean flow such that there is exponential growth in $\boldsymbol{k}$ and pseudomomentum $\mathbf{p}$, which are indicated by the large arrows. At the same time, the mean-flow response return flow at second order in wave amplitude (illustrated by the stippled stream lines) acts to squeeze the dipole and thereby reduces its impulse. It can be shown that vortex impulse change and wave pseudomomentum changes balance each other.

References

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[2] Bühler, O., McIntyre, M. E.: Remote recoil: a new wave–mean interaction effect. *J. Fluid Mechanics*, **492**: 207–230, 2003.
[3] Bühler, O., McIntyre, M. E.: Wave capture and wave–vortex duality. *J. Fluid Mechanics*, **submitted 2004**.