XXI ICTAM, 15–21 August 2004, Warsaw, Poland

VISCOUS EDDY STRUCTURES IN AN OSCILLATING CYLINDER WITH SHARP CORNERS

M. Branicki and H. K. Moffatt

Department of Applied Mathematics and Theoretical Physics,
University of Cambridge,
Wilberforce Road, Cambridge CB3 0WA, UK

Summary The two-dimensional, unsteady flow of a viscous fluid contained in a cylinder which is subjected to rotational oscillatory motion is considered. It is assumed that the cross-section of the cylinder has sharp corners. The problem is linearised assuming sufficiently small amplitude of oscillations - $\mathrm{Sr} \gg 1$ - and sufficiently large viscosity - $\mathrm{Re} \ll \mathrm{Sr}$. A sufficient condition for the appearance of oscillatory corner eddy structures is obtained.

We consider the two-dimensional motion of a viscous, incompressible fluid contained in an infinitely long cylinder whose cross-section D has sharp corners but is otherwise arbitrary. The flow is driven by rotational oscillations of the cylinder with angular velocity $\Omega = \hat{\Omega}\, e^{i\omega t}$. In the frame of reference rotating with the cylinder, the non-dimensionalised Navier-Stokes equation can be written as

$$\frac{\partial \nabla^2 \psi}{\partial t} - \mathrm{Sr}^{-1}\frac{\partial(\psi, \nabla^2\psi)}{\partial(x,y)} - \mathrm{Re}^{-1}\nabla^4\psi = i\, e^{it}\,, \quad \psi = \frac{\partial \psi}{\partial n} = 0 \ \text{ on } \ \partial D\,, \tag{1}$$

where the Poincaré-type force $\dot{\Omega} \sim i\, e^{it}$ provides a uniform rate of production of vorticity. In general, there are two independent parameters in the problem: the Strouhal number $\mathrm{Sr} = \omega/2\hat{\Omega}$ which is the ratio of the frequency of oscillations ω and the amplitude of oscillations $\hat{\Omega}$, and the oscillatory Reynolds number defined as $\mathrm{Re} = L^2\omega/\nu$, where L is the characteristic length of the cross-section D and ν is the fluid viscosity.

It is shown that for sufficiently large Strouhal number: $\mathrm{Sr} \gg 1$, $\mathrm{Re} \ll \mathrm{Sr}$, and for certain range of corner angles, there exist very interesting oscillatory eddy structures emerging from the corners. In particular, it is shown that infinite sequences of corner-eddy like structures are present in the instantaneous streamline patterns for N-sided regular polygonal cylinders, provided $3 < N < 11$. An examination of time-evolution of these structures reveals an intricate inertial mechanism of the flow reversal when the cylinder changes its direction of rotation. The appearance of corner eddies in steady Stokes flow problems was considered by Moffatt 1964a,b, 1979 and 1980. Recently, Shankar *et al* 2003 studied certain oscillatory eddy structures in a rectangular cavity with periodically moving lid. However, as regards the infinite sequence of corner eddies, no attempt to determine analytically their existence in an unsteady case was presented.
The presence of the oscillatory eddy structures is determined from separable solutions of the linearised equation (1), where the amplitude $\hat{\psi}$ of the stream function $\psi = \hat{\psi}(x,y)\, e^{it}$ satisfies the inhomogeneous equation

$$\nabla^2\hat{\psi} - (i\mathrm{Re})^{-1}\nabla^4\hat{\psi} = 1\,, \quad \hat{\psi} = \frac{\partial\hat{\psi}}{\partial n} = 0 \ \text{ on } \ \partial D\,. \tag{2}$$

The self-adjointness of (2) greatly simplifies derivation of the Green's function for some simple geometries, and allows, for example, for construction of a global solution in a rectangular domain which is of particular interest. For domains with more complex boundaries, determination of a global solution to (2) becomes very difficult, if not impossible. Therefore, the structure of local solutions near a corner is analysed through comparison of the 'driven' component of the flow and the eigenfunction ingredients of the corresponding homogeneous problem which are inevitably present. Using similar techniques to those described in Moffatt 1964a, it is determined that for corner angles α within the range of approximately $82^\circ < \alpha < 146^\circ$, an infinite sequence of corner eddies of decreasing size and rapidly decreasing intensity will be present in the instantaneous streamline pattern. For $\alpha > 146^\circ$ no such eddy structures exist. For $\alpha < 82^\circ$ there might still be a finite number of eddies, but their presence depends on conditions far from the corner, and cannot be determined from local analysis. Numerical simulations are used to investigate these cases.
Our poster will present more detailed analysis of the problem supported by extensive numerical simulations for various geometries of the boundary. Stirring properties of resulting flows will be also discussed.

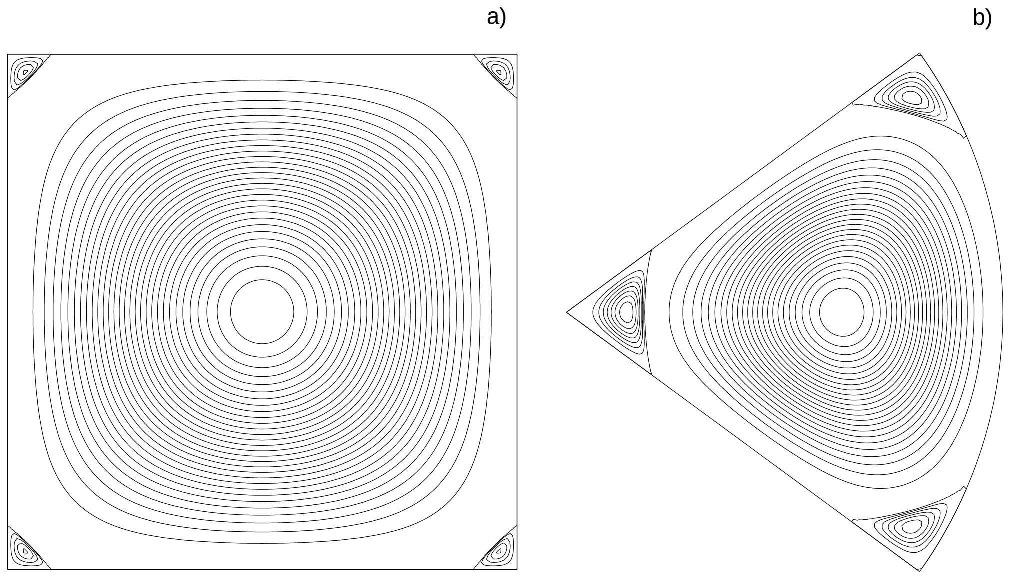

Figure 1. Oscillatory eddy structures in unsteady Stokes flows inside an inifinitely long cylinder subjected to rotational oscillatory motion (in the plane of the figure). Instantaneous streamline patterns are shown in a non-inertial frame oscillating with the cylinder for a) square cross-section, b) section of a circle. Contours represent isolines (not equidistant) of the time-periodic stream function $\Psi = \mathrm{Re}(\hat{\psi} e^{it})$ (see eqn (2)). Only the primary eddies are visualised. The location of the dividing streamlines $\Psi = 0$ changes in time and plays important role during the flow reversal. Streamline patterns for other geometries will be presented and analysed on our poster.

References

[1] Moffatt, H.K. 1964a Viscous and resistive eddies near a sharp corner. J. Fluid Mech. **18**, 1-18.
[2] Moffatt, H.K. 1964b Viscous eddies near a sharp corner. Arch. Mech. Stosowanej **16**, 365-372.
[3] Moffatt, H.K. 1979 The asymptotic behaviour of solutions of the Navier-Stokes equations near sharp corners. *Proc. IUTAM Symp. Approximation Methods for Navier-Stokes Problems. Lecture Notes in Mathematics*, Springer, no. 771,371-380.
[4] Moffatt, H.K. & Duffy, B.R. 1980 Local similarity solutions and their limitations. J. Fluid Mech. **96**, 299-313.
[5] Shankar, P.N. Kidambi, R. Hariharan, J. 2003 Oscillatory eddy structure in a container. J. Fluid Mech. **494**, 163-185.