XXI ICTAM, 15–21 August 2004, Warsaw, Poland

# MASSIVELY PARALLEL SIMULATIONS OF DYNAMIC FRACTURE AND FRAGMENTATION OF BRITTLE SOLIDS

Irene Arias*, Jaroslaw Knap*, Anna Pandolfi**, Michael Ortiz*
*Graduate Aeronautical Laboratories, California Institute of Technology, Pasadena CA 91125, USA
**Dipartimento di Ingegneria Strutturale, Politecnico di Milano, 20133 Milano Italy

*Summary* Massively parallel finite element simulations of dynamic fracture and fragmentation of brittle solids are presented. Fracture is introduced by the adaptive insertion of cohesive elements. The model is validated against specially designed experiments and the crack branching instability is investigated. Mesh sensitivity issues are addressed through the renormalization of the cohesive law.

## INTRODUCTION

We present the results of massively parallel numerical simulations of dynamic fracture and fragmentation in brittle solids. Our approach is based on the use of cohesive models to describe processes of separation leading to the formation of new free surface. Within the framework of the conventional finite element analysis, the cohesive fracture models are introduced through *cohesive elements* embedded in the bulk discretizations. These cohesive elements bridge nascent surfaces and govern their separation in accordance with a cohesive law [1]. In this work we assess the validity of the cohesive models and the computational algorithms. We present careful quantitative validation against experiments designed specifically for this purpose by A. J. Rosakis *et al*. Moreover, the branching instability is investigated numerically. Finally, in relation to the mesh dependency observed for under-resolved meshes, we explore the concept of renormalization of cohesive laws.

## VALIDATION

Xu *et al.* [2] have experimentally investigated the deflection of dynamic mode I cracks at inclined interfaces. Using similar methodology, A. J. Rosakis *et al*. have recently designed a set of well-controlled experiments with the specific goal of testing the fidelity of cohesive fracture models under dynamic conditions. The details of the experimental and selected preliminary results are presented in Fig. 1.

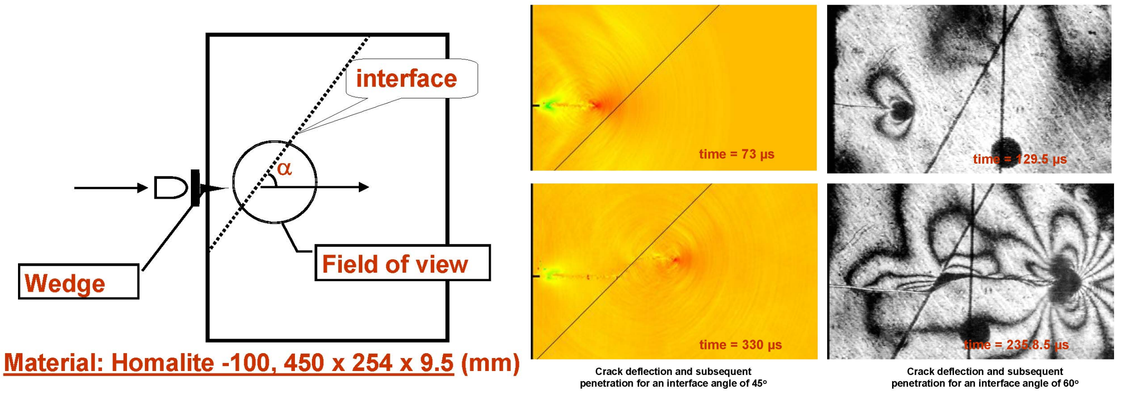


**Figure 1.** Sketch of the experimental set up (left): a pre-notched homalite plate with an inclined interface under a dynamic wedge-loading mechanism, and qualitative comparison of simulation and experiment (right).

## BRANCHING INSTABILITY

Over the last decade, Fineberg and Sharon [3] have conducted a series of well-controlled experiments designed to investigate the dynamic crack propagation in brittle amorphous materials. They have observed that the experimental measurements are in good agreement with the classical continuum theory [4] for low crack velocities. However, the highest crack velocity, they have recorded, is considerably smaller than the predicted asymptotic value $c_R$, i. e., the Rayleigh wave speed of the material. Indeed, the crack behavior goes through a transition when the speed of the advancing crack exceeds the critical velocity $v_c \approx 0.4\, c_R$. Namely, beyond $v_c$, the main crack issues small microscopic side branches. This is a dynamic instability which has a pronounced effect on the structure of the fracture surface. Fineberg and Sharon [3] have made a number of important observations and suggested some universal features of the crack branching instability. We have performed 2D calculations in an attempt to simulate their experiments. A square pre-notched PMMA plate is subjected to an initial uniform strain rate in the vertical direction. Fig. 2 shows three snapshots of the fracture process, in which crack instability, followed by branching and fragmentation are well captured. The main features of the phenomenon pointed out by Fineberg and Sharon (onset of branching, branching patterns), are investigated.

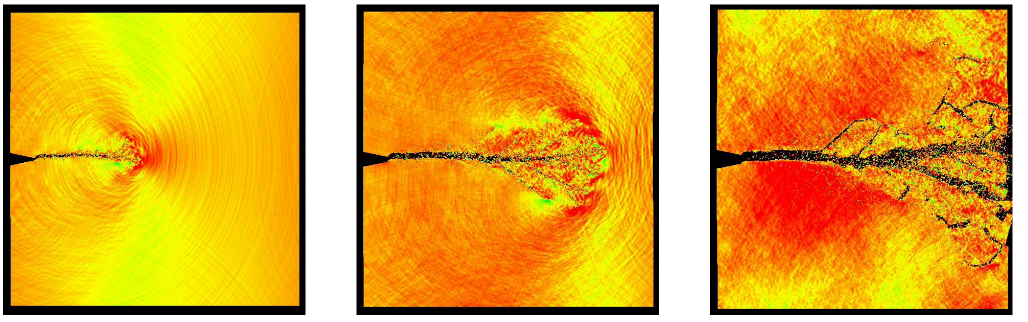

**Figure 2.** Snapshots (increasing times to the right) of the fracture process of a square pre-notched PMMA plate is subjected to an initial uniform strain rate in the vertical direction.

## RENORMALIZATION

As previously noted, simulations concerned with materials possessing as small characteristic length scale may require a large number of elements in the finite element discretization. By way of example, simulations of the Rosakis *et al*. experiment may require as many as 50 million elements. This level of resolution is often prohibitively expensive even when advanced distributed computing environments are available. An alternative approach is to compensate for the lack of resolution by a suitable renormalization of the cohesive law. Specifically, we apply the scaling proposed by Nguyen and Ortiz [5], which relates the effective cohesive energy of a material layer to the interplanar potential under the assumption of nearest-neighbor interactions. A generalization of this renormalization which accounts for arbitrary interactions and surface relaxation has been proposed by Hayes *et al.* [6]. A rigorous mathematical proof of the universality of the renormalized cohesive law has been given by Braides *et al.* [7].

We assess the feasibility of this approach numerically in a double-cantilever-beam test configuration [4] and present preliminary results. Specifically, we endow the cohesive elements with a cohesive law scaled according to the mesh size. Thus, the cohesive strength scales as $1/\sqrt{h}$ and the critical opening displacement scales as $\sqrt{h}$, where $h$ is the characteristic mesh size. The resulting renormalized cohesive law represents the effective behavior of a layer of material of thickness $h$, and has the property that the corresponding cohesive length is automatically resolved by the mesh. In addition, the fracture energy remains invariant under the renormalization. Fig. 3(a) shows the position of the crack tip as a function of time for several under-resolved meshes. The strong mesh sensitivity is apparent in this figure. However, the solutions for these meshes nearly collapse into one curve when renormalized cohesive laws are used, as shown in Fig. 3(b).

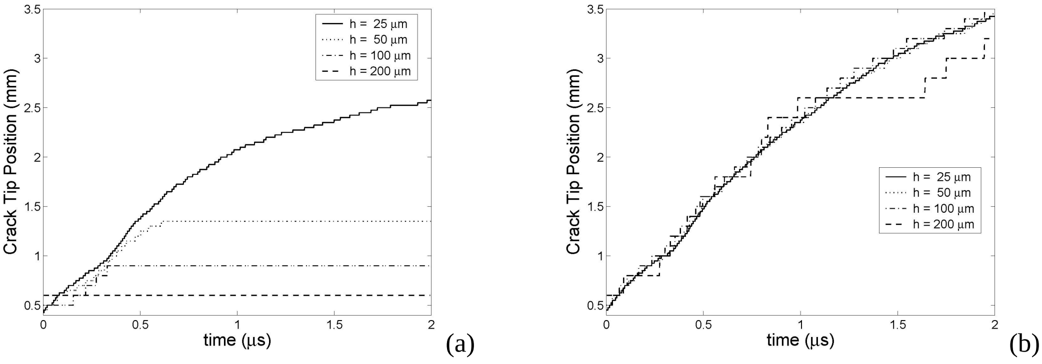


**Figure 3.** Double-cantilever beam test: solutions for under-resolved meshes with actual (a) and renormalized (b) cohesive parameters.

## References

[1] Camacho G.T., Ortiz M.: Computational modeling of impact damage in brittle materials. *Int. J. Solids Struct.* **33**:2899–2938, 1996.
[2] Xu L.R., Huang Y.Y., Rosakis A.J.: Dynamic crack deflection and penetration at interfaces in homogeneous materials: experimental studies and model predictions. *J. Mech. Phys. Solids* **51**:461–486, 2003.
[3] Sharon E., Fineberg J.:Confirming the continuum theory of dynamic brittle fracture for fast cracks. *Nature* **397**:333–335, 1999.
[4] Freund L.B.: Dynamic Fracture Mechanics. Cambridge Univ. Press, NY 1990.
[5] Nguyen O., Ortiz M.: Coarse-Graining and Renormalization of Atomistic Binding Relations and Universal Macroscopic Cohesive Behavior. *J. Mech. Phys. Solids* **50**:1727–1741, 2002.
[6] Hayes R.L., Ortiz M., Carter E.A.: Universal Binding Energy Relation for Crystals, 2004 (submitted for publication).
[7] Braides A., Lew A. J., Ortiz M.: Effective cohesive behavior of layers of interatomic planes, 2004 (in preparation).