# FAST MULTIPOLE ALGORITHM FOR WAVE RESPONSE ANALYSIS OF FLOATING STRUCTURES

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*Summary* The conventional Boundary Element Method (BEM) employing free-surface Green's function is practically impossible to be applied for large-scale analysis such as wave diffraction/radiation analysis for Very Large Floating Structures (VLFS) in general sea-bed topography. This is mainly due to $O(N^2)$ requirement for storage and $O(N^3)$ characteristics for CPU time, where N is the number of unknowns for the BEM. The Fast Multipole Algorithm (FMA) has thus been applied to the BEM utilizing the Green's function by the present authors. The method has O(N) characteristics both for storage requirement and CPU time. The method has been applied to large-scale analysis such as wave response analysis of pontoon type VLFS in general sea-bed topography, and hybrid-type VLFS having complicated shape, where N is the order of $10^4$ - $10^5$.

## INTRODUCTION

For analyzing wave response of floating structures, wave diffraction and radiation forces have to be evaluated. The Boundary Element Method (BEM) employing free surface Green's function is one of the most promising methods which can handle general and complicated shape of floating structures as well as general sea-bed topography.

However, when the methods are applied for the analysis of Very Large Floating Structures (VLFS), the number of unknowns (=N) reaches the order of $10^4$-$10^5$ and thus large storage requirement ($O(N^2)$) and the excessive computation time of $O(N^3)$ (for factorization solvers) make the application of conventional BEM impractical. The precorrected-FFT method [1] has been successfully applied to such a large scale analysis, but not yet for a VLFS in shallow water. We hereby present an alternative approach using Fast Multipole Algorithm (FMA) [2], which has been more commonly used in many fields that would require excessive computation resources.

We have implemented the FMA to our higher-order boundary element program, and examined its efficiency by benchmark calculations. The benchmark calculations include VLFS wave response analysis in variable water depth environment of a real sea [3], and the analysis for a hybrid-type VLFS, which is composed of pontoon part and semi-submersible part [4].

## FORMULATIONS

Consider a general shape VLFS (Very Large Floating Structure) floating in the open sea of variable depth (with constant depth $h$ at infinity). The wetted-surface of VLFS is defined by the symbol $S_H$. The variable depth sea-bottom surface ($S_B$) is defined such that the boundary line ($\Gamma_B$) touches on the flat bottom base-surface of z=-h and the $S_B$ must locate higher than the base surface (z=-h). The coordinate system is defined such that the xy plane locates on the undisturbed free surface and the z-axis points upward.

Consider the long-crested harmonic wave with small amplitude. Assuming the water to be perfect fluid with no viscosity and incompressible, and the fluid motion to be irrotational, then the fluid motion can be represented by a velocity potential $\Phi$. Also, we consider the steady-state harmonic motions of the fluid and the structure, with the circular frequency $\omega$. Then, all of the time-dependent quantities can be represented similarly as follows:

$$\Phi(x,y,z;t) = \mathrm{Re}[\phi(x,y,z)e^{i\omega t}] \tag{1}$$

where $i$ is the imaginary unit ($i^2$=-1) and $t$ is the time.

In this study, we employ the following integral equation as a fundamental equation, which is applicable to floating bodies with variable-depth sea bottoms and/or breakwaters:

$$(4\pi + \int_{S_I} \frac{\partial G_2}{\partial z} dS)\phi(\vec{x}) + \int_{S_H \cup S_B} \{\phi(\vec{\xi})\frac{\partial G}{\partial n} - \phi(\vec{x})\frac{\partial G_2}{\partial n}\} dS - \int_{S_H \cup S_B} G \frac{\partial \phi(\vec{\xi})}{\partial n} dS = 4\pi\phi_I(\vec{x}) \tag{2}$$

Here, $\vec{x} = (x,y,z)$ and $\vec{\xi} = (\xi,\eta,\zeta)$. The symbol $S_I$ designates the inner plane of $z=-h$ inside the boundary $\Gamma_B$. The function $G \equiv G(\vec{x};\vec{\xi})$ represents the Green function representing water waves, and satisfying the boundary conditions on the free surface, on the flat sea bottom ($z=-h$), and the radiation condition at infinity. The series form can be represented as follows:

$$G(\vec{x};\vec{\xi}) = \sum_{m=0}^{\infty} \frac{2K_0(k_m R)}{N_m} \cos k_m(z+h)\cos k_m(\zeta+h) \tag{3}$$

$$N_m = \frac{h}{2}(1 + \frac{\sin 2k_m h}{2k_m h}), \quad k_m \tan k_m h = -K \tag{4}$$

where $k_m$ ($m \ge 1$) is a positive real number, and $k_0 = ik$. The symbol $R$ represents horizontal distance between $\vec{x}$ and $\vec{\xi}$. As an alternative expression of the Graf's addition theorem for the Bessel function, the next equation can be obtained.

$$K_0(k_m R) = \sum_{n=-\infty}^{\infty} K_n(k_m r)e^{in\theta} I_n(k_m \rho)e^{-in\Phi} \tag{5}$$

Here, $\vec{x}=(r,\theta,z)$ and $\vec{\xi}=(\rho,\Phi,\zeta)$ are the points measured from newly defined cylindrical coordinate systems, of which origin $O$ may be referred to as the multipole expansion point. The multipole expansion point $O$ can be located arbitrarily on the undisturbed free surface (thus on the $xy$ plane of the global rectangular coordinate system) under the restriction of $\rho<r$. Substituting Eq. (5) into Eq. (3), we have

$$G(\vec{x},\vec{\xi})=\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}M_{mn}K_n(k_m r)e^{in\theta}\cos k_m(z+h) \tag{6}$$

$$M_{mn}=\frac{2}{N_m}I_n(k_m\rho)e^{-in\Phi}\cos k_m(\zeta+h) \tag{7}$$

Thus, we have obtained the expression of the Green function represented in the form of the multipole expansions. Combining the above formula with the acceleration technique using hierarchical algorithm, the acceleration of the HOBEM program has been made. In this study, we have employed the quadrant-tree to define the tree structure of cells and the hierarchical computation algorithm [2].

## NUMERICAL EXAMPLES

The wave response analysis of a box-like VLFS has been made by using the fast multipole method developed in this study and by the program using direct LU factorization solver. The specifications of the VLFS are: the length $L$=1,500m, the beam $B$=150m, the draft $d$=1m, the rigidity as an elastic plate $D$=3.88x$10^7$ kNm, and the Poisson's ratio $\nu$=0.3. Number of modal functions employed is 160 (20 in longitudinal and 8 in beam). The water depth is 8m (constant). The wave period is 18 seconds (the corresponding incident wave length is $\lambda$=156.8m), and the angle of wave incidence is $\beta=\pi/4$. **Table 1** shows the performance of the benchmark calculations, where in the developed program, GMRES solver is used with the residual tolerance $\varepsilon=10^{-4}$. We note that the fast multipole method (FMM) is efficient both in CPU time and memory allocation compared to the direct method using either LU factorization solver or GMRES solver. Next, the wave response analysis of the same VLFS model located in the variable depth sea representing natural reef has been made. The variable depth surface ($S_B$) is discretized into 20,278 elements of the size of 12.5mx12.5m, and the number of nodes is 61,721. For meshing the VLFS model, the model B (number of nodes is 5,377) in **Table 1** is used; thus the total node number of the analyzed model becomes 67,098. The computation time was 38.4 hour using 5 CPUs for the wave period of 18 seconds, the wave direction of $\beta=\pi/4$, and the residual tolerance $\varepsilon=10^{-2}$. **Fig. 1** shows the deflection amplitudes in the variable depth sea. **Fig. 2** shows a snapshot of the surface elevation around the floating body, where the diffraction and the radiation waves are both included. Results for a hybrid-type VLFS will be presented in the Congress.

**Table 1 Benchmark calculations by FMA and direct method.**

| Model | Typical panel size | Number of nodes | Fast multipole method | | Direct method | | |
|---|---|---|---|---|---|---|---|
| | | | CPU time | Memory allocation | CPU time: *LU* factorization | CPU time: GMRES solver | Memory allocation |
| A | 25.0m | 1,609 | 1.8 min | 27 MB | 1.55 min | 3.28 min | 54 MB |
| B | 12.5m | 5,377 | 10.4 min | 89 MB | 28.5 min | 36.4 min | 489 MB |
| C | 6.25m | 19,393 | 77.4 min | 315 MB | (907 min) | (474 min) | (6 GB) |
| D | 3.125m | 73,345 | 775 min | 1.15 GB | (708 hr) | (113 hr) | (85 GB) |
| E | 1.5625m | 284,929 | 1689 min | 1.75 GB | (1658 days) | (71 days) | (1.2 TB) |

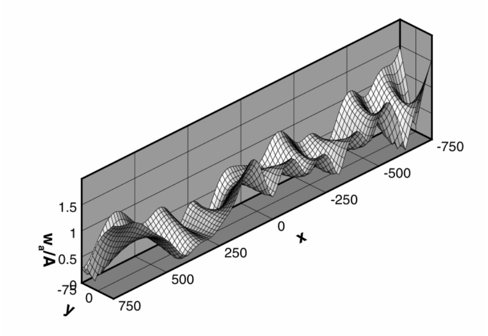


**Fig. 1 Deflection amplitude for Model B in variable depth sea (in the reef sea).**

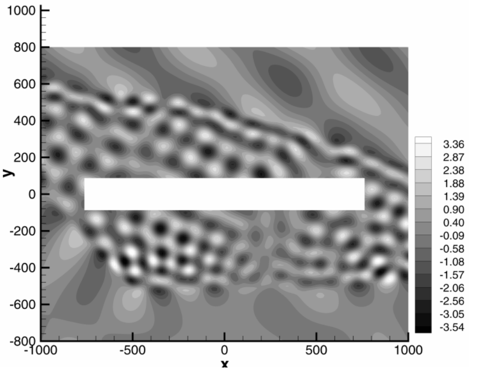


**Fig. 2 Snapshot of the surface elevation around the VLFS in the reef sea.**

## References

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