MULTISCALE BUCKLING ANALYSES OF CORRUGATED FIBERBOARD

Hirohisa Noguchi *, Nobutada Ohno **, and Dai Okumura **
* Department of System Design Engineering, Keio University, Kohoku-ku, Yokohama 223-8522, Japan
** Department of Mechanical Engineering, Nagoya University, Chikusa-ku, Nagoya 464-8603, Japan

Summary In this paper, multiscale micro-macro interaction analyses of corrugated fiberboard are conducted by the finite element method in conjunction with the homogenization theory. The updated Lagrangian method is adopted for the geometrically nonlinear analyses of microstructure and the scaled corrector method is adopted for detecting the buckling mode. By comparing with the experimental results, the validity of the proposed method is illustrated.

INTRODUCTION

Corrugated fiberboard is frequently used as packing material and, being piled together, it is also utilized as shock absorber because it can assimilate impact energy by large deformation of inner corrugating medium. The mechanical properties of corrugated fiberboard are sometimes unstable due to buckling and very difficult to estimate because of the variety and the complexity of microscopic structure. Macroscopic behavior of corrugated fiberboard is also varied by the change of mechanical properties of microscopic structures undergoing large deformation. In this paper, firstly a focus is placed on the variation of the macroscopic property of corrugated fiberboard caused by the microscopic inner local buckling. Assuming that a piled corrugated fiberboard is composed of periodic structures, the macroscopic equivalent properties are calculated from the microscopic, properties and deformation by using the updated Lagrangian geometrically nonlinear finite element method combined with the homogenization theory. Secondly, the micro-macro interaction analysis based on the homogenization theory is carried out. Macroscopic buckling induced by microscopic buckling is demonstrated and the results are compared with the experimental results.

MICRO-MACRO INTERACTION ANALYSIS BASED ON THE HOMOGENIZATION [1,2]

The rate form of principle of virtual work is written at time t, as

$$\int_{{}^t v} {}_t^t\dot{S}_{ij}\delta_t E_{Lij} d^t v + \int_{{}^t v} {}^t T_{ij}\left(\delta_t E_{NLij}\right)' d^t v = \delta\dot{R} \tag{1}$$

$$\delta_t E_{Lij} = \frac{1}{2}\left(\delta u_{i,j} + \delta u_{j,i}\right) \tag{2}$$

$$\left(\delta_t E_{NLij}\right)' = \frac{1}{2}\left(\delta u_{k,i}\cdot\dot{u}_{k,j} + \dot{u}_{k,i}\cdot\delta u_{k,j}\right) \tag{3}$$

S_{ij} is the Truesdell rate, E_{ij} the Green-Lagrange strain tensor and T_{ij} is the Cauchy stress tensor. The rate of displacement can be divided into $\dot{u}_i^0$ and $\dot{u}_i^*$ as shown in Eq.(4), which are macroscopic and microscopic components, respectively.

$$\dot{u}_i = \dot{u}_i^0 + \dot{u}_i^* \tag{4}$$

Substituting Eq.(4) into Eq.(1), Eq.(2) and Eq.(3), the following microscopic and macroscopic equations are finally obtained.

$$\int_Y \left({}_t^t C_{ijpq} + \delta_{pi}\,{}^t T_{qj}\right)\chi_{p,q}^{kl}\delta u_{i,j}^* dY = \int_Y \left({}_t^t C_{ijkl} + \delta_{ki}\,{}^t T_{lj}\right)\delta u_{i,j}^* dY \tag{5}$$

$$\int_{{}^t v} D_{ijkl}^{\mathrm{H}}\,{}^t\dot{u}_{k,l}^0\delta u_{i,j}^0 d^t v + \int_{{}^t v}\delta_{ki}\,{}^t T_{lj}^{\mathrm{H}}\,{}^t\dot{u}_{k,l}^0\delta u_{i,j}^0 d^t v = \delta\dot{R}^{\mathrm{H}} \tag{6}$$

where D_{ijkl}^{H} is the homogenized stiffness tensor. T_{lj}^{H} is the homogenized Cauchy stress tensor. ${}_t^t C_{ijkl}$ is the constitutive tensor. χ_i^{kl} is the characteristic displacement vector corresponding to ${}_t\dot{\varepsilon}_{kl}^0$ as shown in Eq.(7).

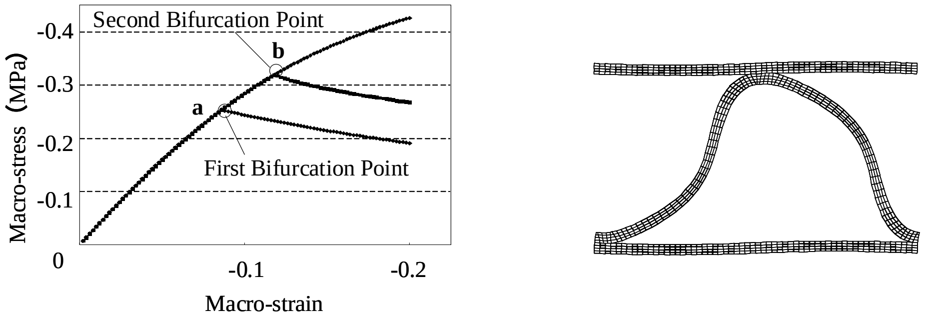
$${}^t\dot{u}_i^* = -\chi_i^{kl}\,{}_t\dot{\varepsilon}_{kl}^0 \tag{7}$$

After finite element discretization, Eq.(5) and Eq.(6) are solved alternately by exchanging the macroscopic strain rates and the homogenized stiffness and the Cauchy stress tensor.

NUMERICAL EXAMPLES

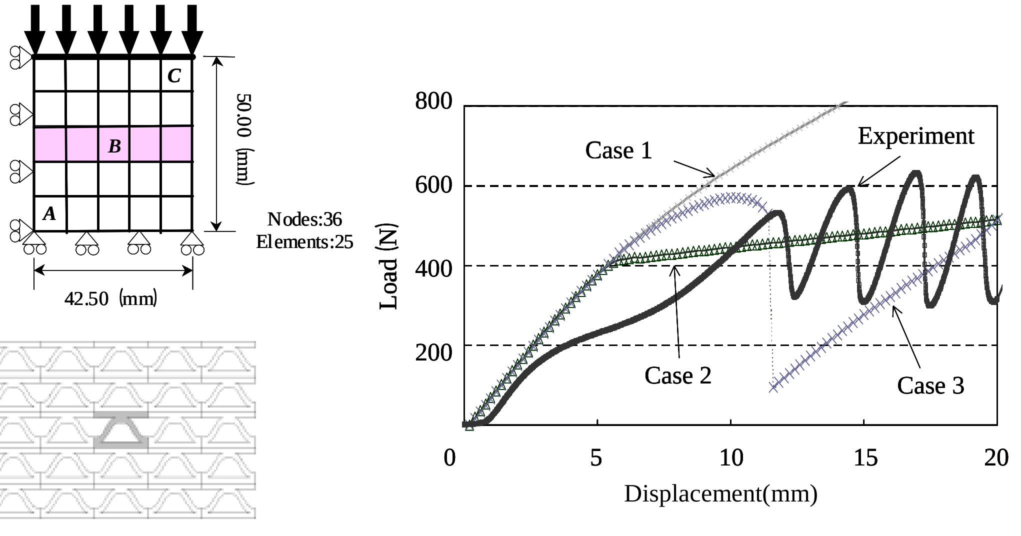
The multiscale micro-macro interaction analyses of corrugated fiberboard are conducted. Fig. 1(a), (b) show an obtained homogenized stress-strain curve and bifurcation buckling mode [3] of microscopic structure, respectively. It is noted that this macroscopic stress-strain curve shows strain-softening behavior and it might cause macroscopic instability. The macroscopic structure is illustrated in Fig. 2(a) and each element in macroscopic structure has the same unit cell shown. In the analyses, the following three cases are considered. Case 1: Buckling (in unit cell) does not occur in all macroscopic elements. Case 2: Buckling occurs simultaneously in all macroscopic elements. Case 3: Buckling occurs only in the middle layer of macroscopic elements.

The load-displacement curves are shown in Fig. 2(b) In the experiment, only one layer buckles first, then other layers buckle subsequently one by one. Each local maximum point in Fig. 2(b) corresponds to buckling of each layer. Which layer starts buckling depends on initial irregularity of corrugated fiberboard. On the contrary, in numerical analysis, load-displacement path can be controlled by path switching in bifurcation analysis. The load-displacement curve obtained in Case 1 is the primary path of this structure and no unit cells buckle. In Case 2 that unit cell is controlled to buckle simultaneously at every point of macroscopic structure, the secondary path varies linearly after bifurcation and go between local maximum and minimum points. The total absorption energy seems equivalent to that by the experiment. In Case 3, only the middle layer is controlled to buckle. As shown in Fig. 2(b), the first buckling point is almost equal to that observed in the experiment, however, the recovery of stiffness is slower than the experimental result. This is due to the ignorance of contact condition inside the corrugated fiberboard. From these results, the validity of the present analyses is clarified.



(a) Homogenized Stress-Strain Curve (b) Bifurcation mode at a

Figure 1. Microscopic response of Corrugated Fiberboard



(a) Analysis model (b) Load-Displacement Curve

Figure 2 Macroscopic response of Corrugated Fiberboard

CONCLUSIONS

In this paper, the multiscale micro-macro interaction analysis of corrugated fiberboard based on the homogenization theory is conducted. Firstly, the microscopic bifurcation of unit cell is analyzed and the macroscopic mechanical properties of corrugated fiberboard are investigated. Secondly, the results of micro-macro interaction analysis are compared with the experimental results and the present analyses are validated.

References

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